Why People Choose Negative Expected Return Assets - An Empirical Examination of a Utility Theoretic Explanation

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Abstract

Using a theoretical extension of the Friedman and Savage (1948) utility function developed in Bhattacharyya (2003), we predict that for assets with negative expected returns, expected return will be a declining and convex function of skewness. Using a sample of U.S. state lottery games, we find that our theoretical conclusions are supported by the data. Our results have external validity as they also hold for an alternative and more aggregated sample of lottery game data.

Keywords: Lotteries, Skewness, Negative Expected Return Assets

JEL Classifications: C51, C52, D11, D12, D80,D81

Why People Choose Negative Expected Return Assets - An Empirical Examination of a Utility Theoretic Explanation

Abstract

Using a theoretical extension of the Friedman and Savage (1948) utility function developed in Bhattacharyya (2003), we predict that for assets with negative expected returns, expected return will be a declining and convex function of skewness. Using a sample of U.S. state lottery games, we find that our theoretical conclusions are supported by the data. Our results have external validity as they also hold for an alternative and more aggregated sample of lottery game data.

Utility theory has been the cornerstone for explaining economic choices under risk and for understanding pricing in the asset market. Utility functions commonly used are increasing and concave functions of wealth. Concave utility functions are standard building blocks for developing the theory of decision making under risk (for example see Huang and Litzenberger (1988) and Ingersoll (1987)). Increasing and concave utility functions reflect diminishing marginal utility of wealth and thus imply that the agent is risk adverse.

State lotteries are a class of product in which participation cannot be explained by the assumption of risk aversion since a risk averse agent will not

buy an actuarially fair lottery ticket, let alone a lottery ticket with a negative

expected return. On average, a lottery ticket returns about 50 cents to players

for a $1 wager. This relatively large negative expected return does not appear to

have reduced participation in state lotteries, however. For example, in the

United States, 41 states and the District of Columbia offer state lotteries. Fiscal

year 2004 sales totaled $48.5 billion ($184 per capita) and net lottery revenue

(sales minus prize payouts, retailer commissions, and administrative costs) to

the states amounted to $13.5 billion, or roughly 1.3 percent of total state

revenue in 2004.[1]

It is common that an individual will display simultaneous risk averse and

risk seeking behavior. For example, the same agent might purchase insurance

(which is risk averse behavior) and purchase lottery tickets (which is risk

seeking behavior). Friedman and Savage (1948) posited that in order to

incorporate simultaneous risk aversion and risk seeking by economic agents,

the utility function of wealth should consist of a concave segment, followed by

a convex segment, followed yet again by a concave segment. However, Quiggin

(1991) has shown that the conclusion of Friedman and Savage (1948) about the

third concave segment in the utility function is erroneous. The utility function

of an economic agent showing simultaneous risk aversion and risk seeking

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[1] Source: National Association of State and Provincial Lotteries
(www.naspl.org). Commerical and Native American casinos generated roughly
$44 billion in revenue and parimutuel wagering revenues total $3.8 billion in
2003 (see www.americangaming.org).

behavior would therefore consist of a single concave segment and a single convex segment.

Bhattacharyya (2003) put forward the conjecture that the utility function of an economic agent is both concave and convex. But, unlike Friedman and Savage (1948) who assume the wealth level of an individual determines whether the agent will act in a risk averse or a risk seeking manner, the utility function proposed by Bhattacharyya (2003) is concave for wealth below the current wealth of the agent and it is convex above the current wealth of the agent.[2] The shape of Bhattacharyya's utility function is drawn in Figure 1.

<<*Figure 1 about here*>>

An agent with such an utility function will simultaneously be a risk averter as well as a risk seeker. Bhattacharyya (2003) finds that for an agent

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[2]Bhattacharyya's (2003) conjecture is inspired by Friedman and Savage (1948)but there is an important difference between Bhattacharyya's (2003) conjecture and that of Friedman and Savage (1948). In Friedman and Savage (1948) some distinct wealth levels are associated with the concave section of the utility function while some other wealth levels are associated with the convex section of the utility function. For an economic agent in Friedman and Savage (1948), the wealth level determines whether the agent will act in a risk averse manner or in a risk seeking manner. In Bhattacharyya (2003), the economic agent will always display risk averse behaviour for wealth below the current wealth and will always display risk seeking behaviour for wealth above the current wealth, i.e., the proposed utility function is always concave for wealth below the current wealth and is always convex for wealth above the current wealth. As a numerical example, suppose we somehow determine that the inflexion point in a Freidman-Savage utility function is at $10 million. In Bhattacharyya (2003), the inflexion point will be at the current wealth level of the economic agent and will be at different wealth levels for agents with different endowments.

with such a utility function, the boundary of the opportunity set in the expected return-skewness space is concave and downward sloping in equilibrium.

In this paper we extend the theory developed in Bhattacharyya (2003) to theoretically characterize the opportunity set for lotteries in the expected return - skewness space and empirically test this theory using two different data sets on U.S. state lottery games. We find that for lottery games, the boundary of the opportunity set in the expected return-skewness space is downward sloping and convex. The relationship is robust across the two data sets.

The paper is organized in the following manner. In section 1, we briefly review the literature. Next, we extend the theory as enunciated in Bhattacharyya (2003) and derive the shape of the opportunity frontier in the expected return-skewness space when expected return is negative. In section 3, we describe the data and develop the research design based on our theoretical extension and then discuss our expected empirical findings. We discuss the results in section 4. In section 5, we apply our theory to a smaller and more aggregated data set in order to examine the robustness of our findings and also to establish the external validity of our results. Section 6 concludes.

1. Literature Review

An extensive literature exists that attempts to explain why risk averse individuals participate in unfair gambles such as lottery games. The Friedman and Savage (1948) model discussed earlier suggests that risk averse people may indulge in unfair gambles if winning will significantly improve their standard of living. Hartley and Farrell (2002) challenge the detractors of the Friedman and Savage (1948) model (e.g. Bailey, Olson, and Wonnacott (1980)) and rigorously show that the cubic utility functions can indeed explain why risk averse individual participate in unfair bets. In a different approach, Kahneman and Tversky (1979) suggest that players place decision weights on the probabilities of each outcome. An over-weighting of low probabilities, especially those associated with multi-million dollar jackpots, may explain the attractiveness of state lotteries. Quiggin (1991) uses a rank-dependent utility function to explain why individuals play lottery games. He theorizes that it is utility maximizing to play the lottery if smaller prizes are offered besides the jackpot. Golec and Tamarkin (1998) argue that bettors' behavior at horse tracks can be explained by expected utility functions that not only consider the mean and variance (risk) of returns, but also the skewness of returns.[3] Bettors are thus risk-averse, but are attracted to the positive skewness of returns

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[3] The work of Golec and Tamarkin (1998) has its basis in what is called the "long shot bias" in horse or dog racing, where high-probability, low variance bets provide relatively high average returns, and low-probability, high variance bets provide relatively lower average returns.

offered by low probability, high variance bets. Garrett and Sobel (1999) extend the work of Golec and Tamarkin (1998) to the case of lottery tickets. They find empirical evidence that lottery players are risk averse but favor positive skewness of returns.

2.   Extending the Theory

Bhattacharyya (2003) uses reductio ad absurdum to develop his theory about the shape of the boundary of the opportunity set for assets in the expected return- skewness space. The basic argument is illustrated in Figure 2.

<<Figure 2 about here>>

The negatively sloped lines are the indifference curves in the expected return-skewness space. Utility increases as the indifference curves increase in the north-easterly direction. A negatively sloped boundary for the opportunity set allows the individual to hold the optimal asset at the tangency point of the indifference curves with the boundary. Note that such tangency is not feasible with any other shape of the boundary. This has been Bhattacharyya's (2003) argument in theorizing about the shape of the boundary.

Bhattacharyya's (2003) argument has been developed in the space where the expected return is positive. In the present paper we are dealing with lottery games which have negative expected returns. Lottery games will generally be negative expected return instruments to players because state governments wish to generate revenue by selling lottery tickets. In order to understand the

shape of the opportunity boundary for lotteries in the expected return-skewness space, we can conceptualize an agent as having two portfolios; one consisting of positive expected return assets and the other consisting of negative expected return assets. Therefore, we need to consider both the positive and negative return directions in the expected return skewness space. This is done in Figure 3.

<<Figure 3 about here>>

We can see from Figure 3 that when the expected return in negative, the shape of the boundary of the opportunity set in the expected return-skewness space will be a negatively sloped convex curve. In equilibrium, the agent will hold a portfolio of positive expected return assets and a portfolio of negative expected return assets. The portfolio of positive expected return assets will be the point where the indifference curve is tangent to the opportunity frontier. The portfolio of negative expected return assets (e.g. lottery tickets) will be the point where the *same* indifference curve intersects the opportunity frontier. The indifference curve thus has two common points with the opportunity frontier - a tangency point in the positive expected return space and an intersection point in the negative expected return space. In the current context where we are discussing negative expected value assets like lottery games, the testable conclusion is that for these assets the opportunity frontier in the expected return-skewness space is a negatively sloped convex curve.

3. Data and Research Design

We estimate several regressions to investigate whether that, for state lotteries, the opportunity frontier in the expected return-skewness space is a negatively sloped convex curve. In other words, we test whether the expected return from lottery tickets is a declining and convex function of the skewness of prize distributions. We should find that, for our sample of lottery games, the slope of this function is negative and the second derivative of the function is positive, thus reflecting the convexity of the opportunity set.

To test our hypothesis, we obtained lottery game data from Kearney (2005). Data include the skewness, expected return, variance, and other game characteristics of 91 lottery games from 33 states over the period 1992 to 1999, obtained on a weekly basis. The data provided by Kearney (2005) are for on-line lotto games.[4]

Our empirical model is:

$$
\begin{aligned}
\text{Expected Return} = {} & \beta_0 + \beta_1 \ \text{Skewness} + \beta_2 \ \text{Skewness}^2 \\
& + \beta_3 \ \text{Variance} + \beta_4 \ \text{Game Type} \\
& + \text{Year and State Dummies} + \tilde{\varepsilon}
\end{aligned}
\tag{1}
$$

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[4] On-line games are those games that require the player to fill out a play slip and watch the drawing on TV. Instant, or 'scratch-off" games, are not included. See Kearney (2005) for a detailed description of the data. Our sample of lottery games has 14,592 oberservations (we omitted missing observations from Kearney's (2005) 15,564 total observations).

Game Type is a dummy variable that has a value of '1' if the top prize of the lottery game is fixed and a '0' if the top prize is parimutuel (e.g. depends upon ticket sales). We include this variable to capture potential differences in the prize structure of lottery games. *Expected Return* is calculated as the expected value minus one. We include state dummy variables to account for unobserved demographic and state-specific lottery game characteristics. The year dummy variables account for unobserved temporal changes that may have influenced the structure of lottery games over time such as increased competition from other lottery games and casino gaming, as well as changes in players' preferences. Descriptive statistics for our key variables are shown in Table 1.

<<Table 1 about here>>

The slope of the curve in the expected return-skewness space is given by

$$\frac{\partial}{\partial \, \text{Skewness}} \text{Expected Return} = \beta_1 + 2\beta_2 \, \text{Skewness} \qquad (2)$$

The second derivative of the function is given by

$$\frac{\partial^2}{\partial \, \text{Skewness}^2} \text{Expected Return} = 2\beta_2 \qquad (3)$$

We therefore predict the following:

$$\beta_1 + 2\beta_2 \, \text{Skewness} < 0 \text{ and } \beta_2 > 0 \qquad (4)$$

4. Results

The results from four different regressions are shown in Table 2. The coefficient on variance is positive and significant, reflecting the familiar risk-return tradeoff. Game type has a positive and significant coefficient, thus providing evidence that lottery games with fixed top prizes have, on average, higher expected returns (about 18 percentage points from model (4)) than lottery games with pari-mutuel top prizes. F-tests (not reported) reveal that the year dummy variables are jointly significant at 1 percent in specifications (2) and (4) and the state dummy variables are jointly significant at 1 percent in specifications (3) and (4).

In accordance with our hypothesis, expected return is a declining and convex function of skewness. The coefficients on skewness and the square of skewness are significant at the 1 percent level. Using the mean value for skewness (reported in Table 2), we find that the predicted first derivative of expected return with respect to skewness is negative (e.g. $\beta 1 + 2*\beta 2*$mean skewness < 0). We also find that the predicted first derivative of expected return with respect to skewness is negative for all lottery games in our sample. Our principle finding that expected return is a declining and convex function of skewness is robust across all four empirical specifications.

<<Table 2 about here>>

5.   Checking for Robustness

We have conducted our investigation based on the data set from Kearney (2005). We have found that our theoretical prediction of expected return for lotteries being a declining convex function of skewness is corroborated by the data. In order to examine the robustness of our findings, we also apply the theory to an empirical test using a different database.

Garrett and Sobel (1999) aggregated data (annual basis) on the expected return and skewness for about 200 U.S. lottery games in 1995. The data set provided by Garrett and Sobel (1999) only allows us to compute the expected value and skewness for the top prize of each lottery game in the sample - information on lower prize tiers is no longer available. Nonetheless, an empirical exercise identical to that presented earlier, but using the data set of Garrett and Sobel (1999) rather than Kearney (2005), will serve as a robustness check on the earlier empirical results. The empirical results using the data set of Garrett and Sobel (1999) are shown in Table 3. As seen in Table 3, the empirical results obtained from using the alternative data set of Garrett and Sobel (1999) also reveal that expected return is a declining and convex function of skewness.

<<Table 3 about here >>

**6.	Conclusion**

In this paper we have developed and empirically tested a theory of the relationship between the expected return and skewness of lottery games. The testable implication of our theory is that expected return from a lottery game is a decreasing and convex function of the skewness of the lottery game. Our empirical results support our theory. We also find that our results are both robust and show external validity as they also hold when we test it on a smaller and more aggregated data setGarrett and Sobel (1999).

There is a simple intuitive explanation for our results. Lotteries are instruments with negative expected returns. So when people buy lottery tickets, they are trading off negative expected returns for skewness. Thus, if a lottery game has a larger prize amount, then a buyer will be willing to accept a lower chance of winning that prize. This also explains why we see different lottery games all priced at $1 but each having widely different top prize amounts. So there will be lottery games with top prizes of a few million dollars and there will be lottery games with top prizes of a few thousand dollars. Both of these lottery games will be priced at $1 and both of these lottery games will have buyers because people are trading off expected return for skewness.

Our paper contributes to the literature that investigates gambling as an economic activity. Ali (1977) examined race track betting and concluded that "betting public behaves as risk lovers" (p.805) Golec and Tamarkin (1998) and Garrett and Sobel (1999) disputed this finding and concluded that gamblers

prefer skewness. Our research suggests that the findings of these different researchers can possibly be explained by a simpler framework-namely that gamblers trade-off expected return for skewness. An interesting area of future research would be to examine racetrack betting using our framework.

The trading off of expected return for skewness could also potentially explain the co-existence of lotteries with other forms of gambling with higher payouts. As for example, lotteries have an average payout rate of 50% compared to the average payout rate of 81% for horse racing and 89% for slot machines (Clotfelter and Cook (1991)). Further research is needed to examine whether the trade off between expected return and skewness can explain the co-existence of various gambling products with different payouts.

Donkers, Melenberg, and Van Soest (2001) examined whether variables like age, income and wealth have any impact on the value function and probability weighting function used in Prospect theory. Taking a cue from this research, an interesting direction for future research will be to examine whether the tradeoff of expected return for skewness is influenced by personal and/or demographic attributes.

Our research provides additional justification for using Friedman and Savage (1948) utility functions in the expected utility framework. It should be reiterated here that the testable predictions for this paper were generated using the modification of the Friedman and Savage (1948) type utility function as developed in Bhattacharyya (2003).

A related area of research will be to examine the relationship between expected return and skewness in the stock market. McEnally (1974) found that "(h)igh risk common stocks, it is frequently observed, do not appear to generate returns commensurate with the level of associated risks"(p.199). Possibly the returns can also be better explained by consideration of the trade off between expected return and skewness. According to Bhattacharyya (2003), the expected return for positive expected return assets like stocks, will be a declining and *concave* function of skewness. Such a research can extend the results in Kraus and Litzenberger (1976)

Future research might also extend these results in the domain of another major negative expected return product, namely insurance. Another possibility is to do an experiment to find whether subjects in an experimental setup indeed make choices in line with our predictions in this paper. A third direction would be extending these results to other gaming products such as casino games.

Acknowledgment

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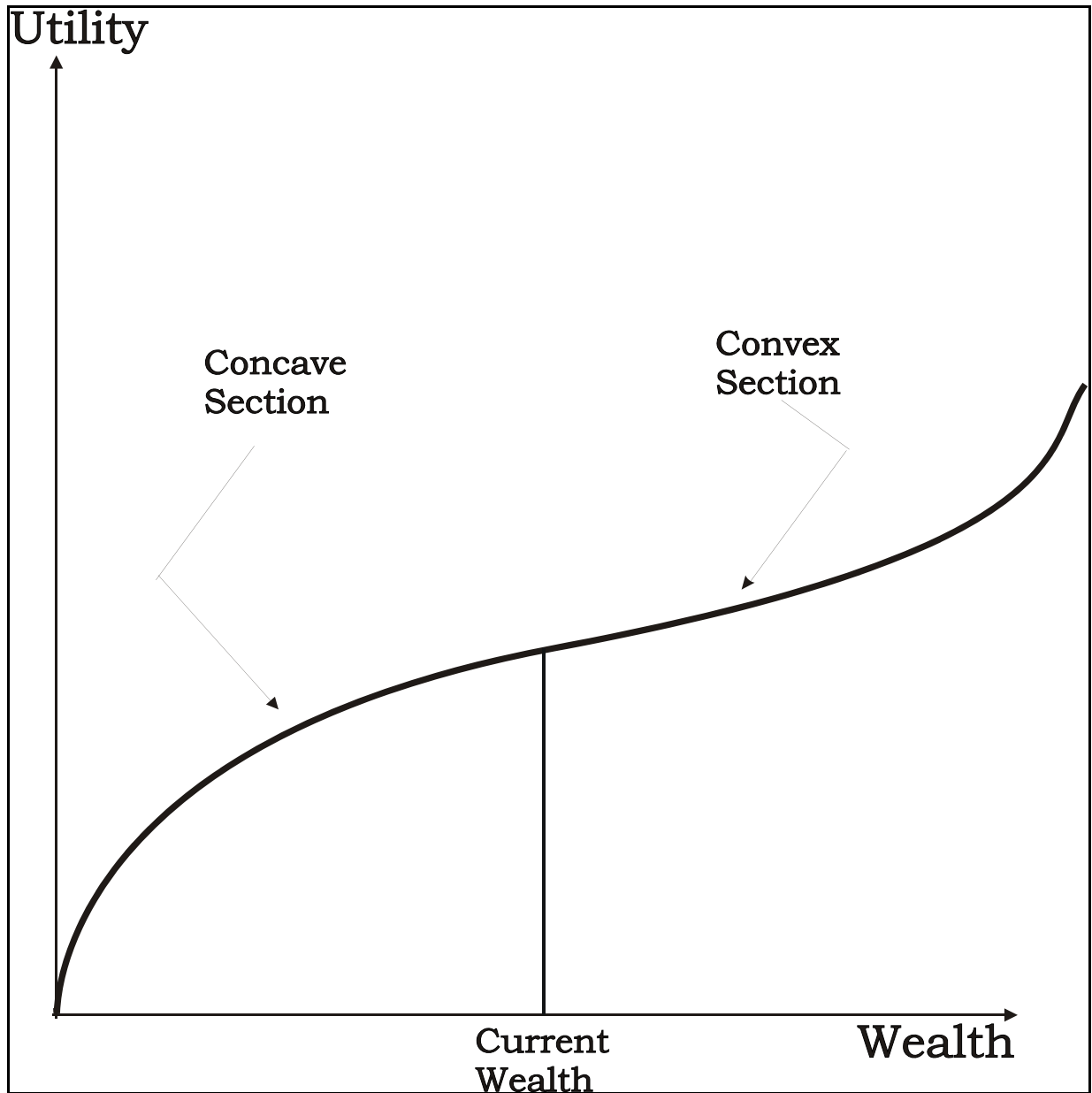
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Figure 1:The Utility Function of Wealth

Note: The utility function proposed by Bhattacharyya (2003) is concave for wealth below the current wealth and is convex for wealth above the current wealth. The utility for the current wealth is at the point of inflexion for the curve.
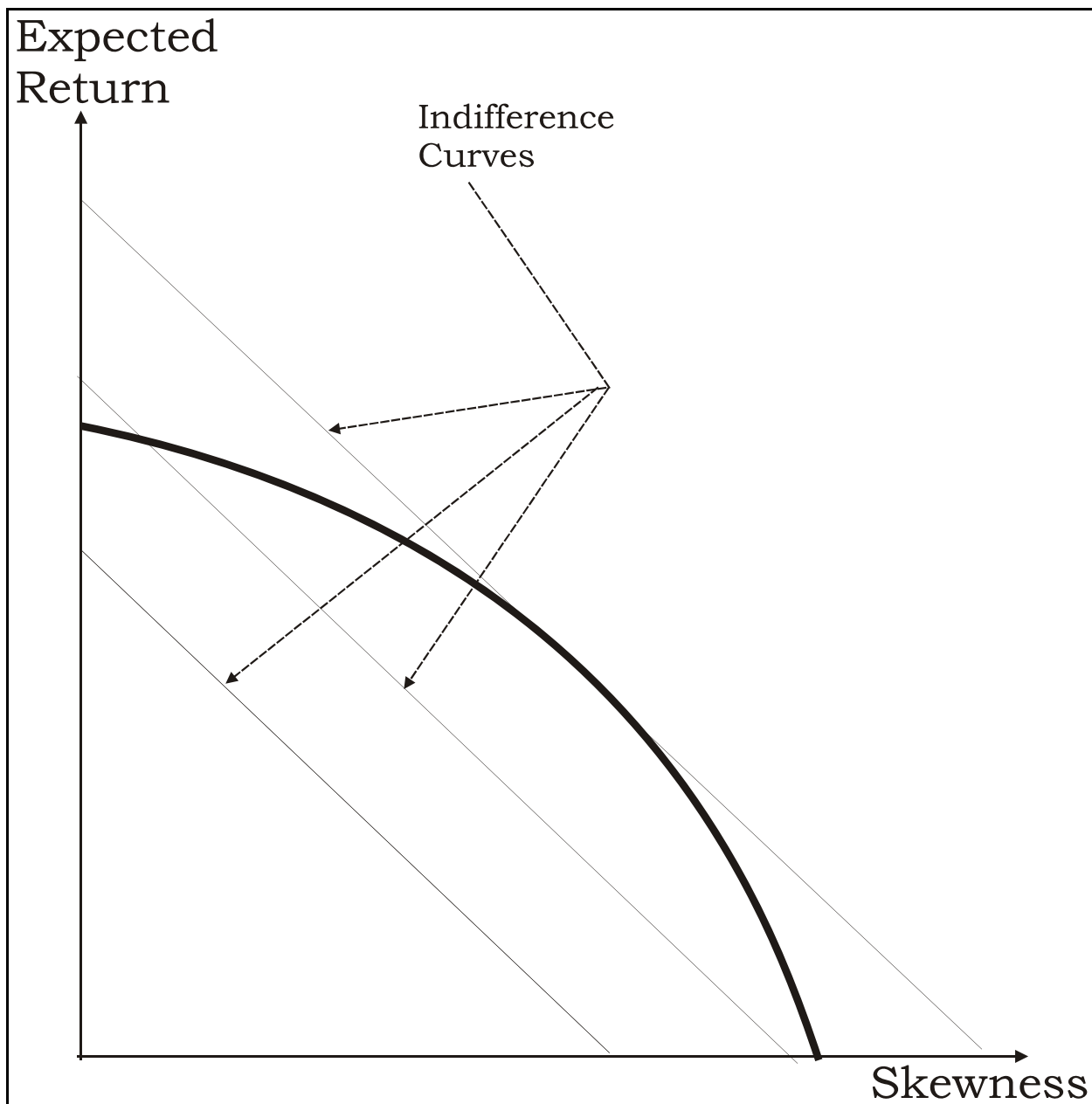
Figure 2:Justification for a Concave Curve as the Boundary of the Opportunity Set in the Expected Return-skewness Space.

Note: The negatively sloped lines are the indifference curves. The thick concave curve is the proposed boundary of the opportunity set. An agent will hold the portfolio where the opportunity set is tangential to the indifference curve as that portfolio position represents the highest attainable utility. We can see that in this case different agents can hold different portfolios in equilibrium because the slopes of the indifference curves can be different for different agents and each agent then can have a distinct optimal portfolio.
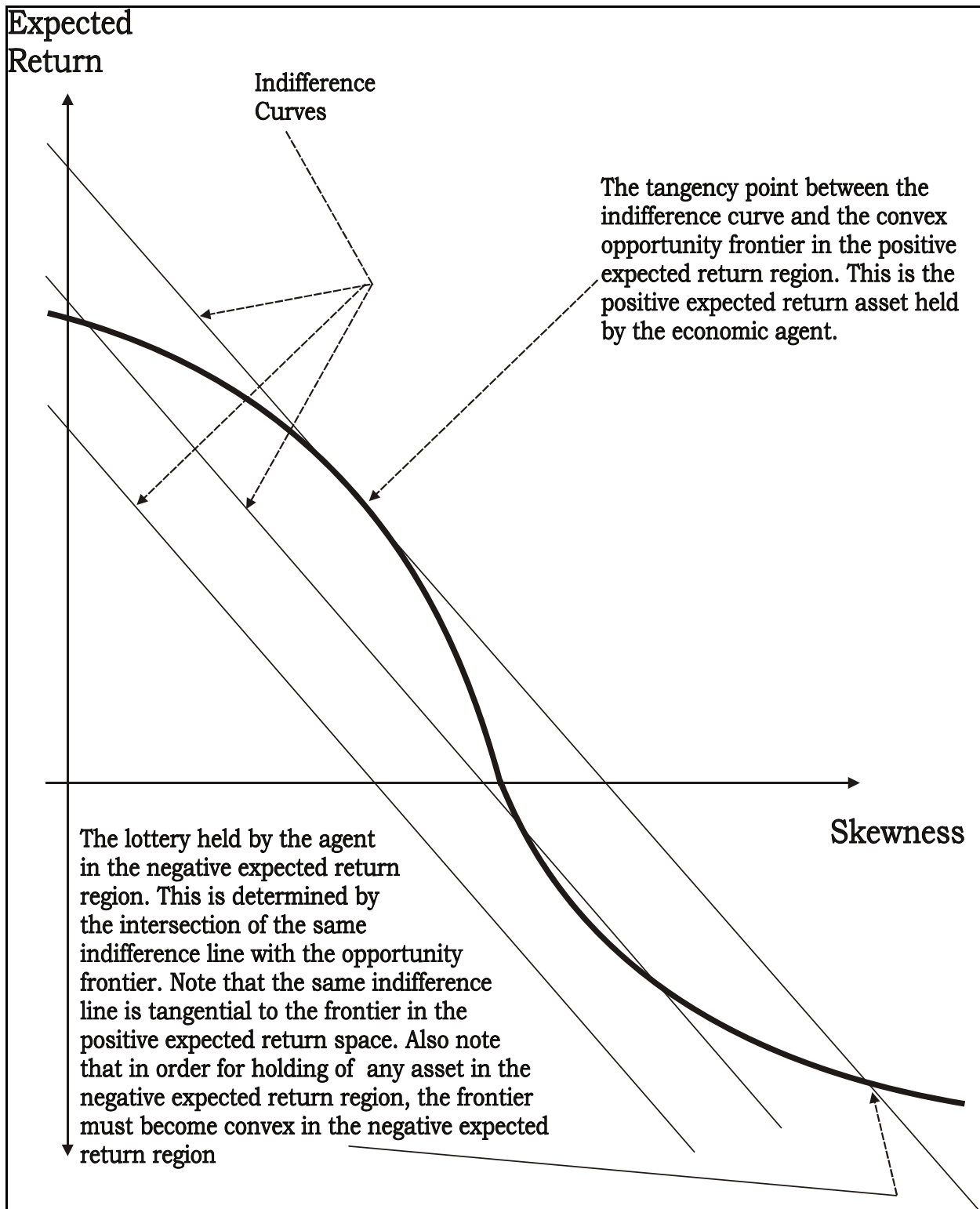
Figure 3: Shape of the Boundary of the Opportunity Set in the Negative Expected Return Space.

Table 1 - Descriptive Statistics

Variable	Mean	Standard Deviation	Minimum	Maximum
Expected Return	-0.4898	0.1367	-0.8767	-0.0044
Skewness	0.3195e+14	0.1511e+15	43,400,000	0.198e+16
Variance	1,204,934	3,624,592	2,087	37,200,000

Number of Observations = 14,592. Data are from Kearney (2005).

Table 2: Regression Results[a]

Variables	(1)	(2)	(3)	(4)
Constant	-0.570*** (0.002)	-0.580*** (0.003)	-0.588*** (0.003)	-0.605*** (0.003)
Skewness (See Note b)	-0.121*** (0.008)	-0.133*** (0.008)	-0.153*** (0.009)	-0.173*** (0.01)
Skewness^2 (See Note c)	0.341*** (0.027)	0.362*** (0.027)	0.396*** (0.027)	0.436*** (0.027)
Variance (See Note d)	0.473*** (0.023)	0.519*** (0.023)	0.608*** (0.024)	0.687*** (0.024)
Game Type (See Note e)	0.113*** (0.002)	0.114*** (0.002)	0.179*** (0.002)	0.187*** (0.002)
Year Dummies	No	Yes	No	Yes
State Dummies	No	No	Yes	Yes
Adjusted R^2	0.200	0.211	0.530	0.546
Observations	14,592	14,592	14,592	14,592
Breusch-Pagan x^2	765.66***	840.33***	6542.55***	6840.35***

Notes:

a). Dependent variable is expected return. Heteroscedasticity-corrected standard errors in parentheses. *** denotes significance at 1 percent, ** at 5 percent, and * at 10 percent. Sample data from Kearney (2005). The Breusch-Pagan x^2 tests Ho: homoscedasticity.

b) Coefficients multiplied by 10^{14}.

c) Coefficients multiplied by 10^{30}.

d) Coefficients multiplied by 10^7.

e) 'Game Type' is a dummy variable having the value of '1' if the top prize is fixed and a '0' if the top prize is pari-mutuel (i.e., a function of ticket sales).

Table 3: Robustness Check Regression Results[a]

Variable	
Constant	-0.752***
	(0.015)
Skewness (See Note b)	-0.222***
	(0.068)
Skewness^2 (See Note c)	0.454***
	(0.121)
Adjusted R^2	0.0376
Observations	209

Notes:

a).   Dependent variable is expected return. Heteroscedasticity-corrected standard errors in parentheses. *** denotes significance at 1 percent, ** at 5 percent, and * at 10 percent. Sample data from Garrett and Sobel (1999). Only data for top prize and the odds of winning for each lottery was available and has been used in this robustness check. Other independent variables used in the main study (e.g., variance, game type, year and state dummies have not been used here because of data non-availability.

b)   Coefficients multiplied by 10^{12}.

c)   Coefficients multiplied by 10^{25}.