Available online at www.sciencedirect.com
SCIENCE DIRECT®
Finance Research Letters 2 (2005) 1–14
Finance Research Letters
www.elsevier.com/locate/frl

# *tay*'s as good as *cay*

Michael J. Brennan [a], Yihong Xia [b,*]

[a] *The Anderson School, UCLA, 110 Westwood Plaza, Los Angeles, CA 90095-1481, USA*
[b] *The Wharton School, University of Pennsylvania, 2300 Steinberg Hall–Dietrich Hall, Philadelphia, PA 19104-6367, USA*

Received 31 August 2004; accepted 18 October 2004

Available online 19 November 2004

**Abstract**

The empirical evidence that the consumption–wealth ratio, *cay*, has strong in-sample predictive power for future stock returns has been interpreted as evidence that consumers take account of future investment opportunities in planning their consumption expenditures. In this paper we show that the predictive power of *cay* arises mainly from a "look-ahead bias" introduced by estimating the parameters of the cointegrating regression between consumption, assets, and labor income *in-sample*. When a similar regression is run, replacing the log of consumption with an inanimate variable, calendar time, the resulting residual, which we label *tay*, is shown to be able to forecast stock returns as well as, or better than, *cay*. In addition, both *cay* and *tay* lose their out-of-sample forecasting power when they are re-estimated every period with only available data.


It is now widely accepted that aggregate security returns contain predictable components. Proposed predictors of returns include interest rates (Lintner, 1975; Fama and Schwert, 1977), the market dividend yield (Campbell and Shiller, 1988; Fama and French, 1988), the term spread and junk bond yield spread (Fama and French, 1989), and the book-to-market ratio (Kothari and Shanken, 1997). On the other hand, Bossaerts and Hillion (1999) and Goyal and Welch (2003) have cast doubt on the existence of any *out of sample* return predictability.

* Corresponding author.
*E-mail address:* yxia@wharton.upenn.edu (Y. Xia).

1544-6123/$ – see front matter 
doi:10.1016/j.frl.2004.10.001

Most recently, a new variable, $\widehat{cay}$, the deviation of (log) aggregate consumption from its predicted value based on a cointegrating regression between (log) consumption, '$c$', (log) aggregate assets, '$a$', and (log) aggregate labor income, '$y$', has been found to be a stronger predictor of both the real return on stocks and the excess of the return on stocks over the riskless interest rate (Lettau and Ludvigson, 2001; (LL)). This new variable explains around 9% of both real market returns and excess market returns over the period 1952.4 to 1998.3 in predictive regressions using quarterly data. The theoretical justification that is offered for the predictive power of $\widehat{cay}$ is based on the assumption that individuals are able to take account of future (risky) investment opportunities in making their current consumption decisions, which implies that aggregate consumption carries information about future returns. If this indeed were the case, we would expect the $\widehat{cay}$ variable to be able to forecast returns out of sample. In this paper we show that the predictive power of $\widehat{cay}$ is entirely *in-sample* and arises mainly from a "look-ahead bias" that is introduced by estimating the parameters of the cointegrating regression between consumption, assets, and labor income *in-sample*. Consequently, $\widehat{cay}$ has no power to *forecast* returns out of sample, and the in-sample predictive power of this variable cannot be taken as evidence that consumers are able to take account of expected returns on risky assets in making their consumption decisions.[1]

The theoretical framework starts from the log-linearized version of the standard budget constraint relating wealth, consumption, and portfolio returns:

$$W_{t+1} = (1 + R_{w,t+1})(W_t - C_t), \tag{1}$$

where $W_t$ is aggregate wealth at the beginning of period $t$, $C_t$ is consumption, and $R_{w,t+1}$ is the return on aggregate wealth.

Equation (1) can be shown to imply the following *approximate* expression for the log consumption–wealth ratio:

$$c_t - w_t \approx \sum_{i=1}^{\infty} {\rho^i}_w (r_{w,t+i} - \Delta c_{t+i}), \tag{2}$$

where lower case letters denote log variables, $\Delta$ is the difference operator, and $\rho_w$ is the steady state investment ratio, $(W - C)/W$. Taking conditional expectations of both sides of (2) yields:

$$c_t - w_t \approx E_t \sum_{i=1}^{\infty} {\rho^i}_w (r_{w,t+i} - \Delta c_{t+i}). \tag{3}$$

Equation (3) implies that the current (log) consumption–wealth ratio must forecast either future returns on aggregate wealth, or future growth rates in consumption. In order to make Eq. (3) operational it is necessary to replace the unobservable aggregate wealth variables, $w_t$ and $r_{w,t+i}$, with observable proxies. In LL, these variables are *approximated* by

[1] Financial economists are more interested in the *economic* (*out-of-sample*) than the statistical (in-sample) importance of return prediction. Bossaerts and Hillion, and Goyal and Welch (op. cit.) emphasize the distinction between in- and out-of-sample predictability. Lewellen and Shanken (2002) discuss the role of learning in producing in-sample return predictability where there is no out-of-sample predictability.

aggregate assets, $a$, labor income, $y$, and the returns on assets, $r_a$, and on human capital, $r_h$, to yield:

$$c_t - \omega a_t - (1-\omega) y_t \approx E_t \sum_{i=1}^{\infty} {\rho^i}_w \left\{ \left[ \omega r_{a,t+i} + (1-\omega) r_{h,t+i} \right] - \Delta c_{t+i} \right\} + (1-\omega) z_t, \tag{4}$$

where $z_t \equiv E_t \sum_{i=1}^{\infty} {\rho_h}^i (\Delta y_{t+1+i} - r_{h,t+1+i})$. Since all the terms on the right-hand side of (4) are assumed to be stationary, $cay \equiv c_t - \omega a_t - (1-\omega) y_t$ is also stationary, so that $c$, $a$, and $y$ must be cointegrated, and *cay* is the deviation from their common stochastic trend. Equation (4) then implies that *cay* must forecast either future market returns or future consumption growth.

## 1. Granger representation and the predictive power of $\widehat{cay}$

The budget constraint, which is the basis for Eq. (3), implies the forecastability of either future asset (human capital) returns $r_a$ ($r_h$) or future consumption growth $\Delta c$, or both, by *cay*. From an empirical point of view, the predictive relation between *cay* and $r_a$ ($r_h$) or $\Delta c$ is established by the Granger Representation Theorem (GRT). If $c_t$, $a_t$, and $y_t$ are cointegrated *and* the vector, $\mathbf{x} = [c, a, y]'$, can be represented as a non-stationary $p$th order vector auto regression (VAR), then the GRT states that there exist parameters $B$ relating the change in the vector of consumption, wealth and labor income, $\Delta \mathbf{x}$, and the one-period lagged values of the cointegration residuals, $\mathbf{z}$:

$$\Delta \mathbf{x}_t = \zeta_1 \Delta \mathbf{x}_{t-1} + \cdots + \zeta_p \Delta \mathbf{x}_{t-p} + \boldsymbol{\alpha} - \mathbf{B} \mathbf{z}_{t-1} + \boldsymbol{\varepsilon}_t. \tag{5}$$

Therefore, whether the variables $c$, $a$, and $y$ are cointegrated and whether the cointegration residual *cay* has forecasting power for $r_a$, and then *in turn* for $r_{\text{S\&P 500}}$, are empirical questions.

The within-sample estimates $\widehat{cay}$ do not seem to forecast growth in labor income or growth in consumption in the sample of 1952.4 to 2000.4, but there is weak evidence that $\widehat{cay}_t$ helps forecast growth in wealth:[2]

$$\Delta a_{t+1} = \underset{(1.88)}{-0.19} + \underset{(1.96)}{0.32} \widehat{cay}_{t-1}, \qquad R^2 = 0.037, \tag{6}$$

[2] There is a subtle timing issue: while $c_t$ and $y_t$ are flow variables for period $t$ which are reported at the *end* of period $t$, $a_t$ is asset wealth *at the beginning of period* $t$. The GRT implies that $cay_t$ forecasts $\Delta a_{t+1}$, and in turn the corresponding stock return, but $\Delta a_{t+1} \equiv a_{t+1} - a_t$, as well as the corresponding stock return, is the change between the beginning of period $t$ and the beginning of period $t+1$ (or, equivalently, the end of period $t$). Since the calculation of $cay_t$ requires information on $c_t$ and $y_t$ which is available only at the end of period $t$, a forecast of $\Delta a_{t+1}$ from $cay_t$ is not feasible because $\Delta a_{t+1}$ is realized *before* $cay_t$ can be calculated. Therefore, the GRT itself does not imply that real-time knowledge of *cay* would allow one to forecast future changes in wealth or its corresponding stock returns. Empirically, the variable calculated at the end of period $t-1$ or beginning of period $t$, $\widehat{cay}_{t-1}$, can also help forecast $\Delta a_{t+1}$ (and in turn the real stock return) in sample, so we report this feasible predictive regression in Eq. (6), even though the GRT does not stipulate that the twice-lagged cointegrating residual must forecast the growth rate of at least one of $c$, $a$, or $y$.

where $\Delta a_{t+1} \equiv a_{t+1} - a_t$ and $a_t$ is measured at the beginning of period $t$. Since growth in wealth is highly correlated with stock returns for the same period,

$$\underset{}{\Delta a_{t+1} =} \underset{(5.78)}{0.004} + \underset{(13.94)}{0.201} r_{\text{S\&P 500},t}, \qquad R^2 = 0.718, \tag{7}$$

results from Eq. (6) thus seem to suggest that $\widehat{cay}$ can *in turn* weakly forecast stock returns.

LL estimate a cointegrating regression of $c$ on $a$ and $y$, using data from the whole sample period, and obtain $\widehat{cay}$ as the residual from this regression. When quarterly S&P 500 Index real returns or excess returns are regressed on the lagged value of $\widehat{cay}$, the regression is highly significant, the corrected $t$-statistic on $\widehat{cay}$ being in excess of 2, and the $R^2$ being around 9%. This predictive relation is far stronger than those obtained previously for other predictors such as the dividend yield or term spread. It is also surprising in view of the lack of success of professional fund managers in timing the market despite the expenditure of millions of dollars on research (Philips et al., 1996). In addition, the strong predictive power of $\widehat{cay}$ for future *stock* returns, which is only indirectly implied by the GRT, dominates in statistical significance its predictive power for future *asset* returns, which as shown in Eq. (6) is only marginally significant despite the fact that the relation between $\widehat{cay}$ and future *asset* returns is directly implied by GRT. This suggests that the forecasting power of $\widehat{cay}$ for *stock* returns is much more than a mere statistical consequence of the GRT.

While LL's findings can be interpreted simply as another piece of empirical evidence of time variation in stock returns and in-sample return predictability, it is important to understand why such a strong predictive relation exists and whether it is genuine or simply a statistical artefact. The interpretation given in LL is that the finding is consistent with optimization by consumers, who seek to smooth consumption, and anticipate future changes in asset values when making consumption decisions. If this is indeed the reason for the strong predictive power of *cay*, then the findings of LL have important implications. First, they imply that the representative consumer has good information about future excess returns despite the fact that attempts to find timing ability among professional investment managers have largely failed. Secondly, the $R^2$ of around 9% implies that a high proportion of the variation in excess returns is due to variation in *expected* returns, which has important implications for the volatility of asset prices.[3] Thirdly, as LL point out, the results have the important policy implication that large swings in the prices of houses and financial assets need not be associated with large movements in consumption since the wealth effect of asset prices on consumption may be muted by changes in investment opportunities. Finally, the important role that they find for $\widehat{cay}$ as a predictor for the investment opportunity set points to the need to take account of time-variation in investment opportunities in asset pricing models.

[3] As Cochrane (1991) points out, excess volatility is the other side of the coin to time varying expected returns.

## 2. Comparison of *cay* and *tay*

Since neither the budget constraint nor the GRT *per se* provide an economic or statistical rationale for the strong predictive power of $\widehat{cay}$ relative to that of other variables that have been analyzed, it is important to assess the robustness of LL's results and to consider whether the reported statistical significance overstates the economic importance of the predictive relation.

There are several possible interpretations of LL's results other than the one given above. One possibility is that exceptionally high consumption (in relation to wealth) leads to exceptionally high profits in the future, and that it is the profits that lift stock prices.[4] A second possibility is that business cycle related deviations of the consumption–wealth ratio from its long run level are coincident with business cycle variation in the market risk premium.[5] A third possibility, and the one that we shall concentrate on here, is the "look-ahead" bias (ex-post trend fitting) that arises from the fact that the coefficients used to generate $\widehat{cay}$ are estimated using the full data sample.[6]

LL are aware of this potential bias and therefore, in addition to their primary results, they report out of sample tests which compare the forecasting performance of $\widehat{cay}^*$ with that of other predictor variables, where $\widehat{cay}^*$ is the value of *cay* that is estimated using only *prior* data on $c$, $a$, and $y$. Their results are summarized in Table 1. The table shows the proportional reduction in the root mean square of the forecasting error of market excess returns when *cay* is included as an additional regressor in the forecasting model (nested comparisons), or when *cay* is used as the predictor in place of the other predictor (non-nested comparisons). The 'Cointegrating vector re-estimated' column refers to the effect of $\widehat{cay}^*$ which is estimated using only prior data, while the 'Fixed cointegrating vector' column refers to the effect of $\widehat{cay}$ which is obtained using the whole sample. It is immediately apparent that the forecasting contribution of $\widehat{cay}$, which is subject to the 'look-ahead' bias, is from 3 to 10 times greater than that of $\widehat{cay}^*$. Thus, on the basis of LL's own analysis, the 'look-ahead' bias does indeed appear to be an important issue.

In order to determine whether or not the forecasting power of $\widehat{cay}$ arises simply because it fits the trend better in the sample, we estimate a simple OLS regression of $t$ on $a$ and $y$,[7] where $t$ is calendar time in months and all standard errors and $t$-statistics are computed

[4] It is possible that the information on consumption and wealth does not become available to the market until the following quarter and that when it is revealed it has a market impact. Huberman and Schwert (1985) report that Israeli index bond prices do not fully reflect recent information about inflation until the official announcement.

[5] Brennan et al. (2004), Fama and French (1989), Keim and Stambaugh (1986), Perez-Quiros and Timmermann (2000), and Whitelaw (1997) all show that the equity premium tends to fall during business cycle expansions and to rise during recessions.

[6] In an independent study, Avromov (2002) also finds that *cay* displays an impressive predictive power only when the shares of asset wealth and labor income (in total wealth) are based on data realized subsequent to the prediction period, and that when constructed using quantities available at the time of prediction, it has poor predictive power and is dominated by traditional predictors such as the book-to-market ratio and the earnings yield.

[7] Data on $c$, $a$, and $y$ come from Sydney Ludvigson's web site http://www.econ.nyu.edu/user/ludvigsons/.

Table 1
One-quarter ahead forecasts using in-sample and out-of-sample estimates of *cay*

| | Cointegrating vector re-estimated (%) | Fixed cointegrating vector (%) |
|---|---|---|
| A. Nested comparison | | |
| 1 $\widehat{cay}_t$ vs. AR | 2.5 | 7.8 |
| 2 $\widehat{cay}_{t-1}$ vs. AR | 1.5 | 4.5 |
| 3 $\widehat{cay}_t$ vs. const | 1.6 | 7.9 |
| 4 $\widehat{cay}_{t-1}$ vs. const | 0.4 | 4.3 |
| B. Non-nested comparison | | |
| 1 $\widehat{cay}$ vs. $r - r_f$ | 2.8 | 9.2 |
| 2 $\widehat{cay}$ vs. $d - p$ | 3.9 | 10.3 |
| 3 $\widehat{cay}$ vs. $d - e$ | 1.8 | 8.3 |
| 4 $\widehat{cay}$ vs. *RREL* | 0.8 | 7.3 |

*Notes*. This table, which is based on LL's Table IV, shows the percentage reduction in the root mean square forecast error of excess returns on the S&P Composite Index as a result of using $\widehat{cay}$ as a predictor. In Panel A the comparison is between a prediction regression with one predictor, either the lagged return (AR) or a constant, and a prediction regression that includes $\widehat{cay}$. In Panel B the comparison is between a prediction regression with the specified regressor ($r - r_f$, $d - p$, $d - e$, *RREL*) and a prediction regression with $\widehat{cay}$ as the predictor. The column labeled 'Cointegrating vector re-estimated' refers to out of sample forecasts in which recursive regressions, using data from 1952.4 to 1968.1, are used to estimate both the parameters in $\widehat{cay}_t$ and the forecasting model each quarter. In the column labeled 'Fixed cointegrating vector,' the cointegrating parameters used to estimate $\widehat{cay}_t$ are set equal to their values estimated in the whole sample.

with correction for heteroscedasticity and autocorrelation:

$$\underset{}{t} = \underset{(51.12)}{-1624.85} + \underset{(9.06)}{85.15a} + \underset{(9.07)}{83.38y}, \qquad R^2 = 0.99. \tag{8}$$

The residual from Eq. (8), $\widehat{tay}$, provides a simple null hypothesis against which to evaluate the $\widehat{cay}$ predictor, for it is clear that, unlike $c$, the simple time trend $t$ only represents an ex-post trend fitting and cannot involve any forecasting or optimization. The residual $\widehat{tay}$ has a correlation of 0.75 with $\widehat{cay}$.

Table 2 reports the estimation results of predictive regressions for the S&P quarterly real return, $r_t$, and the S&P quarterly excess return, $r_t^e$, which is measured relative to the return on a rolled over portfolio of 30-day T-bills. The coefficient of 1.874 on $\widehat{cay}_{t-1}$ reported in column (3) of Panel A compares with a corresponding coefficient of 2.220 reported by LL for a slightly shorter sample period, and the $\bar{R}^2$ of 0.076 compares with their value of 0.09; Panel B contains results for the S&P excess return that are also close to theirs.

The most striking result in Table 2 is that in every case *tay* performs better as a predictor than *cay*. Concentrating on the results in Panel A, the regression using $\widehat{tay}_{t-1}$ has an $\bar{R}^2$ of 0.100, compared with 0.076 for $\widehat{cay}_{t-1}$. When the variables are lagged one more period (regressions 2 and 4) the corresponding $\bar{R}^2$s are 0.077 and 0.043. When the lagged values of both variables are included in the same regression in column (5), $\widehat{tay}_{t-1}$ enters with a $t$-statistics of 2.38 while the $t$-statistics on $\widehat{cay}_{t-1}$ drops to 0.77. The results are similar when two-period lagged values of both variables are included (column (6)). The results for the S&P excess return reported in Panel B of Table 2 are also similar. The overall picture

Table 2
Forecasts of quarterly returns using $\widehat{cay}$, $\widehat{tay}$, and other predictors

A. S&P real return 1952.4 to 2000.4

| | (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) | (9) | (10) |
|---|---|---|---|---|---|---|---|---|---|---|
| constant | 0.010 | 0.011 | −1.138 | −0.882 | −0.383 | −0.053 | 0.010 | 0.011 | 0.011 | 0.011 |
| | (1.98) | (2.11) | (3.19) | (2.41) | (0.75) | (0.11) | (1.73) | (2.02) | (1.76) | (1.96) |
| $\widehat{cay}_{t-1}$ | | | 1.874 | | 0.642 | | | | | |
| | | | (3.24) | | (0.77) | | | | | |
| $\widehat{cay}_{t-2}$ | | | | 1.457 | | 0.103 | | | | |
| | | | | (2.46) | | (0.14) | | | | |
| $\widehat{tay}_{t-1}$ | 0.004 | | | | 0.003 | | | | | |
| | (4.78) | | | | (2.38) | | | | | |
| $\widehat{tay}_{t-2}$ | | 0.004 | | | | 0.004 | | | | |
| | | (4.22) | | | | (2.80) | | | | |
| $\widehat{ta}_{t-1}$ | | | | | | | 0.001 | | | |
| | | | | | | | (2.39) | | | |
| $\widehat{ca}_{t-1}$ | | | | | | | | | 0.026 | |
| | | | | | | | | | (0.27) | |
| $\widehat{ty}_{t-1}$ | | | | | | | | 0.002 | | |
| | | | | | | | | (3.40) | | |
| $\widehat{cy}_{t-1}$ | | | | | | | | | | 0.600 |
| | | | | | | | | | | (2.30) |
| $\bar{R}^2$ | 0.100 | 0.077 | 0.076 | 0.043 | 0.099 | 0.072 | 0.025 | 0.051 | −0.005 | 0.031 |

B. S&P excess return 1952.4 to 2000.4

| | (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) | (9) | (10) |
|---|---|---|---|---|---|---|---|---|---|---|
| constant | 0.010 | 0.010 | −1.107 | −0.805 | −0.503 | −0.114 | 0.010 | 0.010 | 0.010 | 0.010 |
| | (1.94) | (2.06) | (3.24) | (2.36) | (0.99) | (0.25) | (1.74) | (1.98) | (1.76) | (1.94) |
| $\widehat{cay}_{t-1}$ | | | 1.823 | | 0.837 | | | | | |
| | | | (3.28) | | (1.01) | | | | | |
| $\widehat{cay}_{t-2}$ | | | | 1.330 | | 0.203 | | | | |
| | | | | (2.40) | | (0.27) | | | | |
| $\widehat{tay}_{t-1}$ | 0.004 | | | | 0.003 | | | | | |
| | (4.60) | | | | (1.95) | | | | | |
| $\widehat{tay}_{t-2}$ | | 0.003 | | | | 0.003 | | | | |
| | | (3.98) | | | | (2.40) | | | | |
| $\widehat{ta}_{t-1}$ | | | | | | | 0.001 | | | |
| | | | | | | | (2.25) | | | |
| $\widehat{ca}_{t-1}$ | | | | | | | | | 0.040 | |
| | | | | | | | | | (0.40) | |
| $\widehat{ty}_{t-1}$ | | | | | | | | 0.002 | | |
| | | | | | | | | (3.11) | | |
| $\widehat{cy}_{t-1}$ | | | | | | | | | | 0.531 |
| | | | | | | | | | | (2.13) |
| $\bar{R}^2$ | 0.086 | 0.061 | 0.075 | 0.037 | 0.089 | 0.057 | 0.023 | 0.040 | −0.004 | 0.024 |

(*continued on the next page*)

Table 2 (*Continued*)

| C. Real interest rate 1952.4 to 2000.4 | | |
|---|---|---|
| | (1) | (2) |
| constant | −0.0190 | 0.0035 |
| | (0.50) | (4.71) |
| $\widehat{cay}_{t-1}$ | 0.0366 | |
| | (0.60) | |
| $\widehat{tay}_{t-1}$ | | 0.0003 |
| | | (3.35) |
| $\bar{R}^2$ | −0.001 | 0.097 |

*Notes*. The table reports estimates from OLS regressions of stock returns and real interest rates on lagged variables. Variable $\widehat{cay}$ is from LL and variable $\widehat{tay}$ is defined in Eq. (8). Variable $\widehat{ta}$ ($\widehat{ca}$) is the residual from the regression of time (consumption) on a constant and asset wealth. Variable $\widehat{ty}$ ($\widehat{cy}$) is the residual from the regression of time (consumption) on a constant and labor income. The S&P 500 Index real return is constructed as the difference between the logarithm of one plus the nominal return on the S&P 500 Index and the logarithm of one plus the realized inflation rate as measured by the CPI index. The S&P 500 Index excess return is constructed as the difference between the nominal return on the S&P 500 Index and the 30-day T-bill rate. The real interest rate is constructed as the difference between the 30-day T-bill rates and the realized inflation rate. Heteroscedasticity and autocorrelation corrected $t$-ratios are in parentheses.

presented by the table is that, when it comes to prediction, '*tay*'s as good as *cay*', and perhaps a bit better.[8]

Columns (7)–(10) of the table explore the individual roles of $a$ and $y$. $\widehat{ta}$ and $\widehat{ca}$ are the residuals from the regressions of, respectively, calendar time and log consumption on the log of aggregate assets $a$; $\widehat{ty}$ and $\widehat{cy}$ are defined analogously using income $y$ in place of assets $a$. Columns 7 and 9 show that $\widehat{ta}$ retains the in-sample predictive power with a $t$-statistic of 2.39 and an $\bar{R}^2$ of 0.025; on the other hand $\widehat{ca}$ has no predictive power. Thus, even if we were to accept the hypothesis that the consumption–wealth ratio has predictive power for asset returns (where wealth includes human capital as well as financial assets), the same approach shows that the consumption–assets ratio has no predictive power for asset returns.

Column (10) shows that the estimated residual consumption–income ratio $\widehat{cy}$ has predictive power ($\bar{R}^2 \approx 3.1\%$)—high consumption relative to income predicts high future returns. However, it turns out that $\widehat{ty}$ does even better ($\bar{R}^2 \approx 5.1\%$). It seems likely that $\widehat{ty}$ also tracks the business cycle.

Figure 1 plots the time series of $\widehat{cay}$ and $\widehat{tay}$. The current level of $\widehat{tay}$ is just as foreboding as that of $\widehat{cay}$ for returns in 2000 and beyond even though $t$ has no foresight. The reason that both variables are currently negative is that $a$ is well above its historical trend ($y$ is below its historical trend).

Growth in wealth is not only highly correlated with stock returns, it is also positively and significantly correlated with the real interest rate $r_f$, calculated as the difference between

[8] Both Stambaugh (1999) and Amihud and Hurvich (2004) show that the predictive coefficient estimates as reported in Table 2 are biased. We implemented both the Stambaugh bias correction formula and the Amihud–Hurvich bias reduction regression and found that the bias-corrected coefficient estimates are close to those reported in Table 2.

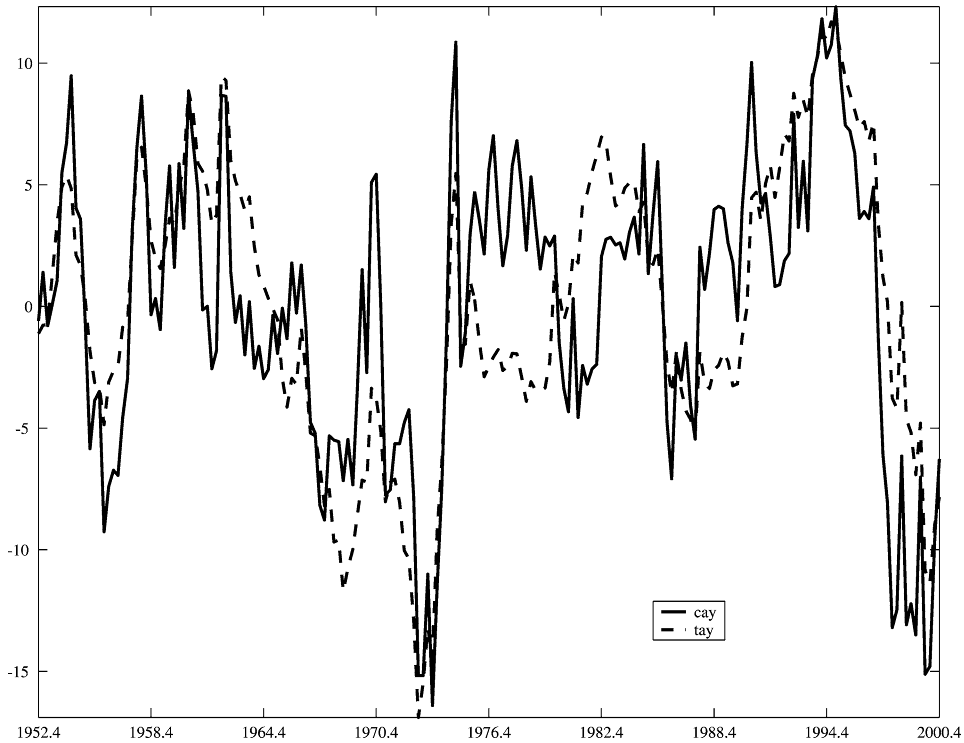


Fig. 1. The figure plots the time series of *tay* and modified *cay*. *cay* is demeaned and multiplied by 500.

the one-month T-bill rate and the CPI inflation rate:

$$\underset{}{\Delta a_{t+1}} = \underset{(2.89)}{0.005} + \underset{(2.68)}{0.649} r_f, \qquad R^2 = 0.045.$$

Since the GRT implies that $\widehat{cay}$ forecasts $\Delta a$ and since $r_f$ is significantly correlated with $\Delta a$, the logic of the forecasting power of $\widehat{cay}$ for stock returns implies that $\widehat{cay}$ also helps forecast the real interest rate $r_f$, which is an element of the individual's future investment opportunity set as well. In Panel C of Table 2, we report the result of regressing the realized real interest rate on $\widehat{cay}_{t-1}$ and $\widehat{tay}_{t-1}$. $\widehat{cay}$ has no predictive power for real interest rates. However, somewhat surprisingly, $\widehat{tay}$ predicts the real interest rate as well as it predicts real S&P 500 Index returns: the $R^2$ for both is around 10%. When $\widehat{tay}$ is high, the real interest rate also tends to be high. Further examination shows that this is because high real interest rates tend to be associated with periods in which labor income, $y$, is below trend.

The results in this section have shown that, when the cointegrating vector is estimated *in sample*, return forecasts constructed from the inanimate variable, calendar time $t$, perform at least as well as those constructed from aggregate consumption, suggesting that the forecasting power of $\widehat{cay}$ is most likely due to the ex post successful fitting of the trend within the sample. In the following section we compare the out-of-sample forecasting power of $\widehat{cay}$ with that of $\widehat{tay}$.

## 3. Out of sample comparisons

Examination of the out-of-sample forecasting performance of $\widehat{cay}$ and $\widehat{tay}$ serves to address the issue of whether the in-sample performance is mainly due to a "look-ahead" bias so that the apparently strong in-sample results are likely to be spurious. In recent years, researchers have cast doubt on the reliability of predictive regressions for stock returns because predictor variables such as the dividend yield, yield spreads and short term interest rates are highly persistent. For example, Ferson et al. (1999) and Torous et al. (2004) argue that persistence in both *expected* stock returns and the predictive variable would cause spurious regressions in which the $t$-statistics and $R^2$ will be biased upwards. The autocorrelation of the *realized* real S&P 500 Index return is only 0.1, but the persistence of the *expected* stock return is not known *a priori*, so that the simulation results in Ferson et al. (1999) are not directly applicable.[9] Out-of-sample forecasting performance provides an alternative check since, if the in-sample predictive relation is spurious or unstable, we should not expect to detect any out-of-sample forecasting power.[10]

In constructing comparisons of the out-of-sample forecasts for S&P 500 Index returns, we broadly follow LL. The first set of forecasts is constructed using values of $\widehat{cay}$ and $\widehat{tay}$ from *fixed* cointegrating vectors that are estimated using data from the whole sample period from 1952.4 to 2000.4.[11] Separate predictive regressions for the S&P 500 Index real and excess returns are estimated using both $\widehat{cay}$ and $\widehat{tay}$ as predictive variables. In addition, we report results for forecasts based on both one and two-period lagged values of the predictors to allow for the possible effect of data publication lags.

In order to construct the out-of-sample forecasts, the predictive regressions are estimated recursively using data from 1952.4 to the quarter immediately preceding the forecast period, and the first forecast period is set at 1968.2. Panel A in Table 3 reports root mean squared errors (RMSE) for the forecasts based on *fixed* cointegrating vectors. The results for the S&P 500 Index real and excess returns are qualitatively similar. Like LL, we find that $\widehat{cay}$ improves on the constant forecast: the reduction in the RMSE is around 1.9% (1.75%) for the real (excess) return.[12] However, we find that $\widehat{tay}$ predicts even better than $\widehat{cay}$: it reduces the RMSE by about 2.1% (1.3%) relative to $\widehat{cay}$ for the real (excess) return. The pseudo $R^2$ is calculated as one minus the squared ratio of RMSE from the predictive regression using $\widehat{cay}$ or $\widehat{tay}$ to RMSE from regression using a constant so that a larger pseudo $R^2$ indicates a larger reduction in the mean square forecast error of the variable relative to the constant forecast. It is around 3.7% (3.5%) when $\widehat{cay}$ is used to forecast

[9] Our own simulation evidence shows that there is virtually no bias in the predictive coefficient estimates for $\widehat{cay}$ or $\widehat{tay}$. In our simulation, stock returns are generated from simulated predictive variable with the same autoregressive coefficient and first two moments as $\widehat{cay}$ (or $\widehat{tay}$) together with the same predictive coefficient as the corresponding $\widehat{cay}$ (or $\widehat{tay}$) predictive coefficient. The simulated stock returns also have the same autoregressive coefficient and the first two moments of the S&P 500 Index return.

[10] It is not uncommon when dealing with stock returns to find that strong in-sample predictive power does not survive out of sample. In a comprehensive study, Bossaerts and Hillion (1999) find strong in-sample but dismal out-of-sample forecasting power of equity returns across nine countries. Their analysis of the power of the test shows that the dismal out-of-sample performance *cannot* be attributed to a lack of power in out-of-sample tests.

[11] Values of $\widehat{cay}$ are taken from Sydney Ludvigson's home page: http://www.econ.nyu.edu/user/ludvigsons/.

[12] LL (2001, Table IV) report a 1.6% improvement for the real return.

Table 3
Root mean square errors and pseudo $R^2$ for out-of-sample forecasts of real returns and excess returns using $\widehat{cay}$ and $\widehat{tay}$ for the period from 1968.2 to 2000.4

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| Panel A. Fixed cointegrating vector | | | | | | | |
| | Constant | $\widehat{cay}_{t-1}$ | | $\widehat{tay}_{t-1}$ | $\widehat{cay}_{t-2}$ | | $\widehat{tay}_{t-2}$ |
| | Root mean square error | | | | | | |
| S&P real return | 0.0837 | 0.0821 | | 0.0804 | 0.0833 | | 0.0809 |
| S&P excess return | 0.0817 | 0.0803 | | 0.0792 | 0.0815 | | 0.0798 |
| | | Pseudo $R^2$ (%) | | | | | |
| S&P real return | | 3.77 | | 7.83 | 1.11 | | 6.59 |
| S&P excess return | | 3.45 | | 5.97 | 0.37 | | 4.66 |
| Panel B. Cointegrating vector re-estimated | | | | | | | |
| | Constant | $\widehat{cay}^{DLS}_{t-1}$ | $\widehat{cay}^{OLS}_{t-1}$ | $\widehat{tay}_{t-1}$ | $\widehat{cay}^{DLS}_{t-2}$ | $\widehat{cay}^{OLS}_{t-2}$ | $\widehat{tay}_{t-2}$ |
| | Root mean square error | | | | | | |
| S&P real return | 0.0837 | 0.0872 | 0.0846 | 0.0851 | 0.0868 | 0.0845 | 0.0840 |
| S&P excess return | 0.0817 | 0.0850 | 0.0828 | 0.0831 | 0.0845 | 0.0827 | 0.0822 |
| | | Pseudo $R^2$ (%) | | | | | |
| S&P real return | | −8.46 | −2.11 | −3.25 | −7.39 | −1.94 | −0.72 |
| S&P excess return | | −8.31 | −2.62 | −3.53 | −7.08 | −2.44 | −1.27 |

*Notes*. The table reports the root mean square errors, RMSE, and pseudo $R^2$ (which is calculated as one minus the squared ratio of RMSE from predictive regression using $\widehat{cay}$ or $\widehat{tay}$ to RMSE from regression using a constant), for out-of-sample one-quarter-ahead forecasts of the real return $r_t$ and the excess return $r_t^e$ on the S&P Composite Index for two different forecasts. The column titled 'Constant' reports the RMSE using the prior sample mean as the predictor. The column titled $\widehat{cay}_{t-1}$ reports the RMSE for a forecast of $r_t$ ($r_t^e$) using $\widehat{cay}_{t-1}$ as a predictive variable where the predictive regression is estimated by ordinary least squares using all the sample data from 1952.4 to the immediately preceding quarter; the column titled $\widehat{cay}_{t-2}$ corresponds to forecasts based on $\widehat{cay}_{t-2}$ as the predictor. The columns titled $\widehat{tay}$ are constructed in a similar fashion. The initial prediction period for the contemporaneous predictors is 1968.2 and the final one is 2000.4; the predictions for the lagged predictors start one (two) period(s) later.

In Panel A, $\widehat{cay}$ and $\widehat{tay}$ are estimated using the whole sample period from 1952.4 to 2000.4 ($\widehat{cay}$ is taken from Sydney Ludvigson's home page). In Panel B, $\widehat{cay}$ and $\widehat{tay}$ are estimated using data from 1952.4 up to the forecast quarter. While $\widehat{tay}$ is estimated using the ordinary least squares, $\widehat{cay}$ is estimated using two different approaches: (1) a dynamic least squares technique with eight leads and lags where all the leads and lags are in the information set at the time of forecast (DLS); and (2) ordinary least squares without any leads and lags (OLS).

the real (excess) return, but it improves to 7.8% (6.0%) when $\widehat{tay}$ replaces $\widehat{cay}$. When the predictors are lagged two periods instead of one, the results are qualitatively similar.

Although the results reported in Panel A are based on recursive regressions, the predictive variable $\widehat{cay}$ ($\widehat{tay}$) is constructed from a dynamic (ordinary) least squares regression that uses future data and hence is subject to a "look-ahead" bias. Therefore, Panel B reports similar forecast comparisons when $\widehat{cay}$ and $\widehat{tay}$ are obtained from regressions that are re-estimated each period using only data prior to the forecast period. While $\widehat{tay}$ is re-estimated using the recursive ordinary least squares regression (OLS), the residual $\widehat{cay}$ is estimated using both an OLS and a dynamic least squares (DLS) technique with eight leads and lags

Table 4
Root mean square errors and pseudo $R^2$ for out-of-sample forecasts of real returns and excess returns using $\widehat{cay}$ and $\widehat{tay}$ for the period from 1976.1 to 2000.4

Panel A. Fixed cointegrating vector

| | Constant | $\widehat{cay}_{t-1}$ | | $\widehat{tay}_{t-1}$ | $\widehat{cay}_{t-2}$ | | $\widehat{tay}_{t-2}$ |
|---|---|---|---|---|---|---|---|
| | Root mean square error | | | | | | |
| S&P real return | 0.0749 | 0.0777 | | 0.0734 | 0.0771 | | 0.0735 |
| S&P excess return | 0.0744 | 0.0772 | | 0.0737 | 0.0766 | | 0.0739 |
| | Pseudo $R^2$ (%) | | | | | | |
| S&P real return | | −7.75 | | 4.01 | −5.93 | | 3.55 |
| S&P excess return | | −7.63 | | 1.88 | −5.96 | | 1.52 |

Panel B. Cointegrating vector re-estimated

| | Constant | $\widehat{cay}^{DLS}_{t-1}$ | $\widehat{cay}^{OLS}_{t-1}$ | $\widehat{tay}_{t-1}$ | $\widehat{cay}^{DLS}_{t-2}$ | $\widehat{cay}^{OLS}_{t-2}$ | $\widehat{tay}_{t-2}$ |
|---|---|---|---|---|---|---|---|
| | Root mean square error | | | | | | |
| S&P real return | 0.0749 | 0.0830 | 0.0789 | 0.0780 | 0.0802 | 0.0773 | 0.0766 |
| S&P excess return | 0.0744 | 0.0826 | 0.0788 | 0.0783 | 0.0795 | 0.0770 | 0.0767 |
| | Pseudo $R^2$ (%) | | | | | | |
| S&P real return | | −22.79 | −11.08 | −8.57 | −14.73 | −6.50 | −4.67 |
| S&P excess return | | −23.19 | −12.15 | −10.52 | −14.04 | −7.05 | −6.09 |

*Notes*. The table reports the root mean square errors, RMSE, and pseudo $R^2$ (which is calculated as one minus the squared ratio of RMSE from predictive regression using $\widehat{cay}$ or $\widehat{tay}$ to RMSE from regression using a constant), for out-of-sample one-quarter-ahead forecasts of the real return $r_t$ and the excess return $r_t^e$ on the S&P Composite Index for two different forecasts. The column titled 'Constant' reports the RMSE using the prior sample mean as the predictor. The column titled $\widehat{cay}_{t-1}$ reports the RMSE for a forecast of $r_t$ ($r_t^e$) using $\widehat{cay}_{t-1}$ as a predictive variable where the predictive regression is estimated by ordinary least squares using all the sample data from 1952.4 to the immediately preceding quarter; the column titled $\widehat{cay}_{t-2}$ corresponds to forecasts based on $\widehat{cay}_{t-2}$ as the predictor. The columns titled $\widehat{tay}$ are constructed in a similar fashion. The initial prediction period for the contemporaneous predictors is 1968.2 and the final one is 2000.4; the predictions for the lagged predictors start one (two) period(s) later.

In Panel A, $\widehat{cay}$ and $\widehat{tay}$ are estimated using the whole sample period from 1952.4 to 2000.4 ($\widehat{cay}$ is taken from Sydney Ludvigson's home page). In Panel B, $\widehat{cay}$ and $\widehat{tay}$ are estimated using data from 1952.4 up to the forecast quarter. While $\widehat{tay}$ is estimated using the ordinary least squares, $\widehat{cay}$ is estimated using two different approaches: (1) a dynamic least squares technique with eight leads and lags where all the leads and lags are in the information set at the time of forecast (DLS); and (2) ordinary least squares without any leads and lags (OLS).

given in Eq. (11) in LL.[13] While the DLS is the theoretically correct approach, $\widehat{cay}^{OLS}$ performs better than $\widehat{cay}^{DLS}$ with a smaller RMSE and a larger pseudo $R^2$. Again, $\widehat{tay}$ performs as well as $\widehat{cay}^{OLS}$ and slightly better than $\widehat{cay}^{DLS}$ as a predictor for both real and excess returns. When the two predictive variables are constructed recursively, however, the forecast power of both variables completely disappears. Neither $\widehat{cay}$ nor $\widehat{tay}$ performs as well as the constant forecast: the RMSE under $\widehat{cay}$ and $\widehat{tay}$ is larger than that of a constant forecast, and the pseudo $R^2$ all become negative.

[13] The DLS approach yields unbiased parameter estimates in constructing $\widehat{cay}$ but substantially reduces the sample size by requiring leads and lags of the first difference term in the regression. The number of observations in the *cay* regression increases from 61 to 193 in the OLS but ranges from 45 to 176 in the DLS.

Table 4 repeats the exercise of Table 3 except that the first forecast period is delayed to 1976.1. A longer sample period should yield better estimates of the cointegrating coefficients, so that extending the first period by eight years should improve the results, especially since the cointegrating coefficients are super-consistent and converge at a rate proportional to $T$ rather than the usual $\sqrt{T}$. Instead, the reverse is found: the pseudo $R^2$ is negative no matter whether $\widehat{cay}$ is estimated using the whole sample or re-estimated using only data prior to the forecast period. The RMSE from the one-period-ahead forecast using $\widehat{cay}$ exceeds that from using a constant by more than 3.8% when $\widehat{cay}$ is estimated using the whole sample and by 5.4% (10.8%) when $\widehat{cay}^{\mathrm{OLS}}$ ($\widehat{cay}^{\mathrm{DLS}}$) is re-estimated using only data prior to the forecast period. Although $\widehat{tay}$ still retains some predictive power when it is estimated using the whole sample, it also completely loses its out-of-sample forecasting power when it is re-estimated every period with only available data.

Consistent with the implications from the analysis in the previous section, results from this section indicate that the cointegration residual has no *out of sample* predictive power for stock returns. Taken together, these results suggest that the strong in-sample predictive power of $\widehat{cay}$ is very likely to be due to the “look-ahead” bias introduced by ex post fitting a trend within the sample. As a by-product of the out-of-sample analysis, we also find evidence that the cointegration structure as well as the predictive regression may be unstable over time, the details of which are analyzed in Hahn and Lee (2001) and are thus omitted from the current paper.

## 4. Conclusion

LL have shown that the consumption–wealth residual helps forecast stock returns and have offered the interpretation that this is due to the ability of the representative agent to forecast future stock returns and to adjust consumption accordingly. In this paper, we have shown that a purely mechanistic variable, *tay*, that is constructed using calendar time in place of consumption, performs as well as, or better than, the consumption based variable, *cay*, in predicting stock returns and real interest rates. The predictive power of both *tay* and *cay* completely disappears when constructed out-of-sample, suggesting that the in-sample predictive power of both variables is highly possibly derived from a successful fitting of the trend in the sample. Thus, the strong empirical results of LL are most likely to be due to a ‘look-ahead’ bias and should be interpreted with caution.

## Acknowledgments

We thank an anonymous referee, Andrew Ang, John Cochrane, Eugene Fama, Robert Hodrick, Ross Valkanov, and especially Amir Yaron for helpful comments and suggestions. We also benefited substantially from Sydney Ludvigson and Martin Lettau’s response to earlier drafts of the paper.

## References

Amihud, Y., Hurvich, C.M., 2004. Predictive regressions: A reduced-bias estimation method. Journal of Financial and Quantitative Analysis. In press.
Avromov, D., 2002. Stock return predictability and model uncertainty. Journal of Financial Economics 64, 423–458.
Brennan, M.J., Wang, A.S., Xia, Y., 2004. Intertemporal capital asset pricing and Fama–French three-factor model. Journal of Finance 59, 1743–1776.
Bossaerts, P., Hillion, P., 1999. Implementing statistical criteria to select return forecasting models. What do we learn? Review of Financial Studies 12, 405–428.
Campbell, J.Y., Shiller, R., 1988. The dividend–price ratio and expectations of future dividends and discount factors. Review of Financial Studies 1, 195–228.
Cochrane, J.H., 1991. Volatility tests and efficient markets. Journal of Monetary Economics 27, 463–485.
Fama, E.F., Schwert, W., 1977. Asset returns and inflation. Journal of Financial Economics 5, 115–146.
Fama, E.F., French, K.R., 1988. Dividend yields and expected stock returns. Journal of Financial Economics 33, 3–27.
Fama, E.F., French, K.R., 1989. Business conditions and expected returns on stocks and bonds. Journal of Financial Economics 25, 23–49.
Ferson, W.E., Sarkissian, S., Simin, T., 1999. Spurious regressions in financial economics? Working Paper. Boston College.
Goyal, A., Welch, I., 2003. Predicting the equity premium with dividend ratios. Management Science 49, 639–654.
Hahn, J., Lee, H., 2001. On the estimation of the consumption–wealth ratio: cointegrating parameter instability and its implications for stock return forecasting. Working Paper. Columbia University.
Huberman, G., Schwert, G.W., 1985. Information aggregation, inflation, and the pricing of indexed bonds. Journal of Political Economy 93, 92–114.
Keim, D., Stambaugh, R., 1986. Predicting returns in the stock and bond markets. Journal of Financial Economics 17, 357–390.
Kothari, S.P., Shanken, J., 1997. Book-to-market, dividend yield, and expected market returns: a time-series analysis. Journal of Financial Economics 44, 169–204.
Lettau, M., Ludvigson, S., 2001. Consumption, aggregate wealth and expected stock returns. Journal of Finance 56, 815–849.
Lewellen, J., Shanken, J., 2002. Learning, asset-pricing tests, and market efficiency. Journal of Finance 57, 1113–1145.
Lintner, J., 1975. Inflation and security returns. Journal of Finance 30, 259–280.
Perez-Quiros, G., Timmermann, A., 2000. Firm size and cyclical variation in stock returns. Journal of Finance 55, 1229–1262.
Philips, T.K., Rogers, G.T., Capaldi, R.E., 1996. Tactical asset allocation: 1977–1994. Journal of Portfolio Management 23, 57–64.
Stambaugh, R.F., 1999. Predictive regression. Journal of Financial Economics 54, 375–421.
Torous, W., Valkanov, R., Yan, S., 2004. On predicting stock returns with nearly integrated explanatory variables, Journal of Business 77. In press.
Whitelaw, R., 1997. Time variation in Sharpe ratios and market timing. Working Paper. New York University.