Stock Option Returns: A Puzzle

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Abstract

Under very weak assumptions, the expected returns of European call options must be positive and increasing in the strike price. This paper investigates the returns to call options on individual stocks that do not have an ex-dividend day prior to expiration. The main findings are that over the 1996 to 2005 period (1) out-of-the-money calls have negative average returns, and (2) the average returns of high strike calls are lower than those of low strike calls. The puzzling returns are robust to a number of variations in methodology, and are not due to a 'peso' problem. Finally, preliminary evidence is presented that is consistent with investor skewness-seeking contributing to the puzzling call returns.

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Introduction

This paper finds that the average returns to out-of-the-money (OTM) stock calls are negative and average call returns are decreasing in the strike price. These findings are surprising, because very weak economic assumptions imply that expected call returns must be positive and increasing in the strike price.

In the framework of the Merton (1973a) continuous-time CAPM and the Black-Scholes formula, the intuition for the result that expected European call returns should be positive and increasing in the strike price is straightforward. Calls are levered positions in the underlying stocks, and higher strike calls are more levered positions. If an underlying stock has positive beta, all calls will have positive betas that exceed the beta of the underlying stock, and call betas will increase in the strike price as the calls get further out-of-the-money (OTM). Hence, all calls will have positive expected returns and the expected returns will be larger for greater strike prices (holding maturity fixed).

Coval and Shumway (2001) extend this reasoning to much weaker assumptions by proving that expected European call returns must be positive and increasing in the strike price provided only that (1) investor utility functions are increasing and concave and (2) stock returns are positively correlated with aggregate wealth. They also show empirically that average S&P 500 index call option returns are indeed positive and increasing in strike price.

This paper shows that individual stock call option returns, by contrast, do not obey even these very weak restrictions. Individual stock options are American, so in general the Coval and Shumway (2001) analysis does not apply to them. The empirical work in this paper, however, is limited to call options whose underlying stocks do not have ex-dividend dates prior to expiration. As proven in Merton's (1973b) seminal paper, it is irrational to exercise such a call option early. Consequently, these calls are effectively European, and their returns should conform to the Coval and Shumway restrictions.

I compute the average return over the remaining life of one month to expiration call options over the January 1996 through June 2005 period. The returns are computed from the call bid-ask midpoints one month before expiration under the assumption that options are held until maturity and exercised if the stock price is greater than the strike price at expiration. I find that high strike calls (those for which the strike price divided by the stock price, K/S, is greater than 1.15) have average returns of −27.84% over the month and that the on average the high strike calls have returns that are 37.90% *lower* than low strike calls (those for which K/S is less than 0.85). When considering these findings, it should be noted that over the data period stock prices rose which biases against finding negative call returns and also against higher strike calls having lower returns than lower strike calls.

These puzzling findings are robust to a number of variations in methodology. Similar results obtain when the calls are assigned to categories based on volatility adjusted moneyness or Black-Scholes delta rather than K/S. Even stronger results are found when the ask price, which may well be a better estimate of the price at which the calls were purchased, is used to compute the returns. The findings are robust to limiting the analysis to calls that actually trade on the day one month before expiration or calls on the largest 200 stocks. In addition, the main findings hold when the sample is split into the period when the stock market bubble of the late 1990s and early 2000s was inflating and deflating and the period after the bubble had deflated. The same pattern of returns is also observed for calls with longer maturity.

Several pieces of evidence also suggest that the negative OTM call returns are due to excessively high call prices rather than the 'peso' problem that a smaller than representative number of large underlying stock returns were realized during the paper's sample period or to jump risk premia that are imbedded in call prices. First, OTM call prices are too high (and, hence, have returns that are too low) relative to call prices that are computed adjusted for realized stock price paths. Second, call returns become positive and increase in the strike price if market call prices are replaced by theoretical call prices from Black-Scholes (1972) model or Merton (1976) jump model where the jump intensity is set to match either the observed jump intensity or three times the jump intensity and the jump risk premia are set to a variety of values. Finally, the sample period from January 1996 to June 2005 used in the analysis has a much higher frequency of large monthly returns than the preceding time period from 1963 through 1995. Consequently, it is unlikely that fewer than expected large returns were realized over 1996 to 2005.

The results presented in this paper are surprising not only because they violate very weak economic assumptions, but also because the empirical option pricing literature suggests that index but not individual stock option prices are problematic. The implied volatilities of at-the-money (ATM) index options are substantially greater than the realized volatility of the underlying index, and the implied volatilities are sharply skewed across moneyness. For individual stock options, on the other hand, implied volatilities are close to realized volatilities, and there is only a mild implied volatility smile across moneyness (Whaley (2003), Bollen and Whaley (2004), Bates (2003), Garleanu, Pedersen, and Poteshman (2006).) Given these findings about index and stock option prices, one might well expect that index option returns would be less well behaved than stock option returns. At least from the point of view of restrictions on returns derived from weak economic assumptions, just the opposite turns out to be the case.

My findings obviously raise the question of why the restrictions on call returns are violated by stock options. Although the goal of this paper is to document rather than to solve this puzzle, I present some preliminary evidence on its source.

One assumption that lies behind the call return restrictions is that stock returns are positively correlated with aggregate wealth. Of course, it is possible that there are a few stocks that violate this assumption. It seems unlikely, however, that very many optioned stocks do not have returns positively correlated with wealth. The second assumption is that investor utility functions are everywhere increasing and concave. This assumption is tantamount to assuming that investors are always risk-averse. Contrary to this assumption, many important economists have posited that risk aversion and risk seeking co-exist (for example, Smith (1776), Friedman and Savage (1948), Markowitz (1952), Kahneman and Tversky (1979,1992), and Barberis and Huang (2005).)

Two straightforward ways in which the utility assumption may be violated that have recently received attention are for investors to seek idiosyncratic skewness (Barberis and Huang (2005)) or idiosyncratic volatility (Ang et. al., (2006)). In order to provide some evidence on whether these types of preferences may contribute to the puzzling behavior of call returns, I measure both the expected idiosyncratic skewness and volatility and the realized idiosyncratic skewness and volatility for call returns across moneyness categories. I find that for both the expected and realized cases idiosyncratic skewness and volatility increase in strike price. These results are consistent with both idiosyncratic skewness and volatility contributing to the anomalous call returns. Since idiosyncratic skewness and volatility are positively correlated, I perform a double sort of call portfolio returns on idiosyncratic skewness and volatility. This analysis indicates that idiosyncratic skewness but not volatility contributes to the puzzling call returns. Hence, there is preliminary evidence consistent with the hypothesis that skew seeking behavior impacts option returns.

My findings are related to a number of studies that examine stock option prices. For example Bakshi, Kapadia and Madan (2003) show that physical kurtosis of stock returns flattens the slope of volatility smiles using options with K/S from 0.90 to 1.1 on the 20 largest stocks from 1991 to 1995. Duan and Wei (2006) use the same data and find that higher levels of systematic risk leads to a higher levels of implied volatility and a steeper slope for the implied volatility smirk. Goyal and Saretto(2006) investigate trading strategies that go long or short straddles based on forecasted changes in volatilities.

My findings are also related to studies on people's preferences toward risks. For example, Blackburn and Ukhov (2006) recover utility function from call options and returns of underlying stocks that compose the Dow Jones Industrial Average, and suggest that preferences proposed by Friedman and Savage (1948), Markowitz (1952) and Kahneman and Tversky (1979,1992) are reflected in the prices of Dow Jones stocks. Post and Levy (2005) suggest that Markowitz (1952) type utility functions, with risk aversion for losses and risk seeking for gains, can capture the cross-sectional pattern of stock returns. Kumar (2005) shows that stocks with small institutional ownership have a negative idiosyncratic skewness premium.

The remainder of the paper is organized as follows. Section I outlines the Coval and Shumway (2001) restrictions on expected option returns. Section II describes the data and presents summary statistics on option trading during the sample period. Section III develops methods for assigning calls to strike groups and calculating option returns. Section IV reports the call return results. Section V describes potential explanations for the puzzling returns, and Section VI concludes.

I. Restrictions on Expected Option Returns

The introduction provided the intuition within a continuous-time CAPM/Black-Scholes world for expected European call returns being positive and increasing in strike price. I now develop the argument from Coval and Shumway (2001) that shows that these restrictions on expected call returns follow from the much weaker assumptions that (1) investor utility is increasing and concave in wealth and (2) stock returns are positively correlated with wealth.

Assume that investors have a utility function that is increasing and concave in wealth and that they maximize utility over beginning and end-of-period wealth such that their problem can be stated as

$$\max \; U(W_0) + \delta \mathrm{E}\left[U(\tilde{W}_T)\right], \text{ where } U' > 0, \;\; U'' < 0. \tag{1}$$

Let S_0 be the initial price of asset i, and S_T be its end of period random payoff. Then

$$S_0 = \mathrm{E}\left[\frac{\delta U'(\tilde{W}_1)}{U'(W_0)} \cdot S_T\right] = \mathrm{E}\left[m \cdot S_T\right], \tag{2}$$

where $m \equiv \delta U'(\tilde{W}_T) \big/ U'(W_0)$ is the stochastic discount factor.

Assume that asset i's return is positively correlated with future wealth $\tilde{W}_T$, then asset i's future payoff S_T is positively correlated with $\tilde{W}_T$. In this case, the stochastic discount factor m will be negatively related to S_T. This fact can be established by taking the derivative of the stochastic discount factor with respect to S_T

$$\frac{\partial m}{\partial S_T} = \frac{\partial m}{\partial \tilde{W}_T} \cdot \frac{\partial \tilde{W}_T}{\partial S_T} = \frac{\delta U''(\tilde{W}_T)}{U'(W_0)} \cdot \frac{\partial \tilde{W}_T}{\partial S_T} < 0 \tag{3}$$

as by assumption $U' > 0, U'' < 0$, and $\dfrac{\partial \tilde{W}_T}{\partial S_T}$>0.

The expected gross return to maturity on a call option with a strike price K on an underlying security whose future price has a distribution $f(\cdot)$ can be expressed as

$$E\left[R_C(K)\right] = \frac{\int_{S_T=K} (S_T - K) f(S_T) \partial S_T}{\int_{m=0} \int_{S_T=K} m(S_T - K) f(S_T, m) \partial S_T \partial m}, \tag{4}$$

where $f(S_T, m)$ is the joint density of the stock price at expiration and the stochastic discount factor. The expected net return, $\mathrm{E}\left[r_C(K)\right] = \mathrm{E}\left[R_C(K)\right] - 1$, can be written as

$$\mathrm{E}\left[r_C(K)\right] = \frac{\int_{S_T=K} (S_T - K)\left[1 - \mathrm{E}\left[m \mid S_T\right]\right] f(S_T) \partial S_T}{\left[\int_{S=K} \mathrm{E}[m \mid S_T] f(S_T) \partial S_T\right]^2}, \tag{5}$$

where $E\left[m \mid S_T\right]$ is the stochastic discount factor conditional on the future stock price S_T.

The derivative of expected call returns with respect to the strike price is

$$\frac{\partial \mathrm{E}\left[r_C(K)\right]}{\partial K} = \frac{-\mathrm{Cov}\left[S_T - K, \mathrm{E}(m \mid S_T) \mid S_T > K\right]}{\left[\int_{S_T=K} (S_T - K) \mathrm{E}\left[m \mid S_T\right] \frac{f(S_T)}{1 - F(K)} \partial S_T\right]^2} > 0, \tag{6}$$

where $F(\cdot)$ is cumulative distribution function of stock price at expiration. If m is negatively related to the stock price at expiration, S_T, for the entire price range, then call option expected returns will increase in the strike price. The stock is equivalent to a call with zero strike price, which implies that all call options will have positive expected returns larger than the expected return of the underlying stock. Hence, if investor utility is increasing and concave and the underlying stock's future payoff is positively correlated with wealth, European call returns are positive and increasing in strike price.

II. Data, Descriptive Statistics, and Sample Selection

The main data used in this paper are from the IVY DB data set from OptionMetrics. The OptionMetrics data contain daily volume, open interest, national best closing bid and ask prices, implied volatility, and option Greeks for all U.S. exchange-traded stock and index options beginning on January 4, 1996. This data set also has the daily prices, returns, and distributions of all U.S. exchange traded stocks. I obtain data on stock data before 1996 and shares outstanding from the Center for Research in Security Prices (CRSP). The main time period for the study is January 4, 1996 to June 31, 2005.

Table I summarizes stock option trading for each calendar year from 1996 through 2004. Stock option trading experienced high growth over this period. The yearly volume and open interest in 2004 are, respectively, 5.29 and 6.14 times greater than in 1996. Stock market volume, in contrast, increased by a factor of only 2.51. Stock option volume increases at a greater rate when stock market is rising than when it is falling. From 1996 to 2000 when the stock market rose sharply, stock option volume increased at an average yearly rate of 37%. On the other hand, when the stock market declined in 2001 and 2002, stock option volume increased at an average rate of only 4%. When the stock market rebounded beginning in 2003, the option trading once again grew rapidly, increasing by 14% and 23% in 2003 and 2004, respectively.

In analyzing stock option returns, I select calls that meet all four of the following conditions. The first condition is that the underlying stock does not have an ex-dividend date during the remaining life of the call. The return restrictions developed above apply to European calls. Stock option calls, however, are American (i.e., subject to early exercise). As shown by Merton (1973b), however, American calls on underlying stocks that do not pay dividends are not optimally exercised prior to expiration and thus are equivalent to European calls. Hence, the return restrictions apply to calls where the underlying stock does not have an ex-dividend date prior to expiration. By choosing this non-dividend restriction, I delete 14% optionable stocks each month on average. The second condition is that the call bid price is strictly greater than $0.125. This criterion follows the standard practice of eliminating very low price options. The third condition is that the calls must obey the arbitrage bound. Finally, on each option expiration date I select calls that expire on the next expiration date (the expiration dates are the Saturdays after the third Friday of every month). Hence, they have four or five weeks to maturity. I choose calls in this way in order to obtain prices from liquid options while at the same time avoiding overlapping data in the returns that I study.[1] All together there are 1,501,276 call options from January 1996 to June 2005, and on average 1855 stocks in each month.

At any moment in time and for any given expiration date, different underlying stocks have varying numbers of calls with different strike prices. Table II summarizes the characteristics of underlying stocks based on the number of call strike prices that their one month maturity calls have on the Fridays before the option expiration dates. Stocks that have only one strike are of the smallest size, lowest stock price, and lowest beta.[2] Half of the stocks have either two or three strike prices. These stocks are of medium size, price, and beta. On an average expiration date, 268 stocks have one month to expiration calls with five or more strike prices. These stocks have the largest size, stock price, beta, and volatility.[3]

III. Methodology

A. Strike Groups

In order to investigate how call returns vary with strike price, I sort the calls into five strike groups. Various underlying stocks at different option expiration dates have varying numbers of one month calls (i.e., varying number of strikes) with varying ratios of strike price to stock price. In addition, the volatilities of these stocks are not the same. As a result, there is not one obvious best way of classifying calls into strike price groups, and I employ the three different methods summarized in Table III. All three methods ensure that the strike prices from calls on a given underlying stock at a particular point in time increase in the strike group number. Consequently, strike group 1 corresponds to calls with the lowest strike prices and strike group 5 corresponds to calls with the highest strike prices.

The first method of assigning strike groups is to use the ratio of strike price to stock price (*K*/*S*). This is perhaps the most common way of defining option moneyness. Specifically, on each month's option expiration day, I divide call strike prices by closing stock prices and then assign calls to strike groups based on the cutoffs listed in Table III. This method normalizes differences in strike prices by stock price levels. The second method of assigning calls to strike groups is to divide $\ln(K/S)$ by the volatility of the underlying stock, σ. Stock i's day t volatility, $\sigma_{i,t}$, is estimated by

$$\sigma_{i,t} = \sqrt{\frac{252}{60}\sum_{j=0}^{59} r_{i,t-j}^2} \tag{7}$$

where $r_{i,t-j}$ is stock i's return on trading day $t-j$. This method takes into account not only the relationship between the strike price and stock price level, but also incorporates the volatility of the stock. Since the maturities of all of the calls are one month, this method of defining strike groups corresponds to volatility adjusted moneyness. The cutoff values for the different strike groups are again listed in Table III. The final method for assigning calls to strike groups is to use the Black-Scholes (BLS) delta. When computing the Black-Scholes delta, equation (7) is used to estimate volatility. This method of assigning strike groups is used by Bollen and Whaley (2004). It simultaneously accounts for different stock price levels, underlying stock volatilities, and any differences in time to maturity across options. Once again, Table III contains the cutoffs for assignments to different strike groups.

B. Call Returns

Call returns are computed for calls bought on one expiration Friday and held until the next expiration date. Hence, the returns are for holding calls for four or five weeks. I use only returns to expiring calls, because (1) the return restrictions derived above apply to calls held to expiration, and (2) if the factors independent of stock market affect both the beginning and the end periods option prices, the returns based on option prices only may cancel these factors out, while returns to expiration can provide a clearer picture whether the options are over or under valued. I use returns from one expiration date to the next (i.e., four or five week returns), in order to have returns that cover the whole sample period without overlapping.

On each month t's option expiration date, I calculate average return for calls in a strike group on a given underlying stock and then average this quantity across underlying stocks. I compute the call returns from the call's bid-ask midpoint except as otherwise indicated. The equation to compute strike group G's call return on month t's option expiration date is

$$r_t^G = \frac{1}{N_t^{stock,G}} \sum_i^{N_t^{stock,G}} r_{t,i}^G , \tag{8}$$

where,

$$r_{t,i}^G = \frac{1}{N_{t,i}^{call,G}} \sum_j^{N_{t,i}^{call,G}} \frac{\max(S_{t+1,i} - K_{t,i,j}, 0)}{C_{t,i,j}} - 1.$$

In these expressions, i indexes underlying stocks that have at least one call in strike group $G \in \{1,2,3,4,5\}$, $N_t^{stock,G}$ is the total number of such stocks at month t, $r_{t,i}^G$ is the average return on stock *i*'s calls in group G, $N_{t,i}^{call,G}$ is the total number of calls that stock *i* has in group *G*, $S_{t+1,i}$ is stock i's closing price on the next option expiration day, $C_{t,i,j}$ is the call bid-ask midpoint price for the *j*th call on underlying stock *i*, and $K_{t,i,j}$ is the strike price of the *j*th call on underlying stock *i*.

As is evident from Table II, not all underlying stocks have calls in every strike group on every expiration date. In order to investigate directly how call returns vary with strike price, I also calculate the call return difference between high and low strike groups that contain calls from the same underlying stocks. In particular, I compute the return difference between strike group G^H and strike group G^L for calls on the same underlying stock by

$$\begin{aligned} & r_t^{G^H} - r_t^{G^L} = \frac{1}{N_t^{stock,G^H,G^L}} \sum_i^{N_t^{stock,G^H,G^L}} (r_{t,i}^{G^H} - r_{t,i}^{G^L}), \\ & G^H \in \{4,5\},\ G^L \in \{1,2\} \end{aligned} \tag{9}$$

where N_t^{stock,G^H,G^L} is the number of stocks which have calls in both strike group G^H and strike group G^L

IV. Call Option Returns

This section of the paper examines the call returns to different strike groups. In the first subsection, I present summary statistics on call returns and underlying stock characteristics. In the second subsection, I investigate whether call returns are positive and increasing in the strike price as required under weak economic assumptions. We will see that both of these requirements are violated. In the third subsection, robustness tests are conducted which indicate that the puzzling call returns do not have their source in options that do not trade, are not limited to small underlying stocks, are present both during the stock market bubble and after the bubble deflated, and are observed in longer maturity calls.

A. Summary Statistics

Table IV reports monthly average summary statistics for calls and underlying stocks for each of the five strike groups. Each month the call beta is given by $\beta_C = \Omega_C \beta_S$, where Ω_C is the option omega defined as

$$\Omega_C = \frac{S}{C} N\left[\frac{\ln(S/K) + (r + \sigma^2/2)\tau}{\sigma\sqrt{\tau}}\right], \tag{10}$$

where N is the standard normal distribution function, τ is the time remaining to expiration, and the volatility σ is estimated from the previous 60 daily returns. The dollar volume is the number of contracts multiplied by the dollar price of each contract using bid-ask mid points. Table IV reports the averages of these quantities across the 113 months in the sample. Likewise, each month the beta and returns are computed for the underlying stocks in each strike group, where the stock betas are estimated from the previous 24 monthly returns. The time series standard deviation, autocorrelation, and skewness are computed from the 113 months of call portfolio returns. The three panels correspond to the three different methods for sorting calls into strike groups.

Table IV indicates that call beta is increasing in the strike price. The increasing call beta is consistent with the theoretical prediction that call returns should increase across strike prices. Indeed, the increase of the average call beta from about 4 to over 10 as one goes from strike group 1 to strike group 5 suggests that call returns should be sharply increasing in strike price. The fact that all of the call betas are large (e.g., compared to stock betas that tend to be near one) is consistent with the theoretical prediction that call returns should be positive. Group 3 (ATM) calls have the highest volume, although the volumes in group 4 and group 5 (OTM) calls are only slightly smaller in number of contracts. Call return standard deviation and skewness are increasing across the strike groups. The monthly autocorrelations are all small and close to zero. The ITM (i.e., lower) groups have the most underlying stocks. This may be due in part to the fact that stock prices were generally rising over the sample period, since existing calls become more ITM when the underlying stocks increase in value.

B. Call Returns

Table V reports the average of the monthly time series of call returns and return differences for the five strike groups. Table V also reports the average of the monthly time series of the difference between group 5 and group 1 calls where each month only calls on underlying stocks that are represented in both groups are used. In order to increase the number of underlying stocks represented, I also report the return differences between groups 4 and 1 and groups 5 and 2, again for each month only using calls on underlying stocks for which at least one call appears in each group. Monthly strike group returns are computed from equation (8) and monthly return differences between groups are computed from equation (9). In both cases, the bid-ask midpoints are used to compute the call returns. Results from computing the returns from the bid or the ask will be presented below. Means and standard *t*-statistics are computed from the time series of 113 monthly average call portfolio returns.

Recall that group 1 contains the lowest strike, most ITM, lowest beta calls, and group 5 contains the highest strike, most OTM, highest beta calls. Under very weak assumptions, the expected returns of all of the calls should be positive and the expected return of any strike group should be higher than all of the lower strike groups. Table V indicates that both of these predictions are violated. Strike groups 4 and 5 both earn negative returns for all three methods of classifying options into strike groups. For group 5 the average monthly return for the *K*/*S* classification method is –27.84% (*t*-statistic –3.70). For the other two classification methods, the average monthly return is less than –25% with a *t*-statistic of less than –3.75. In addition, the call return differences between groups 5 and 1, groups 5 and 2, and groups 4 and 1 are negative and statistically significant for all three classification methods. For example, for the *K*/*S* classification method the average monthly return difference between groups 5 and 1 is –27.90% (*t*-statistic –5.02). These findings that both call return restrictions are violated are puzzling, because the restrictions are derived under the very weak assumptions essentially that investors are risk-averse and stock prices tend to be higher when overall wealth is higher.

C. Robustness Tests

This subsection of the paper presents several robustness checks. The results presented above compute call returns from option price quotations. This seems reasonable insofar as option market rules require that market makers stand ready to buy and sell a certain number of contracts at their quoted bid and ask prices. There is, however, no guarantee that the options are actually trading. Panel A of Table VI reports the results of performing the analysis only on call options that actually traded (i.e., had trading volume of at least one contract) on the initial expiration date on which they are chosen for inclusion. The main features of the previous results are still present. The group 5 options still experience significantly negative returns, and the difference between higher and lower group options are still negative and significant or marginally significant.

The return calculations equal weight stocks with different market capitalizations. It is interesting to see whether the anomalous return differences are present for large capitalization stocks. Panel B of Table VI repeats the analysis on the calls of the 200 largest market capitalization stocks. The returns for group 5 and for the difference between group 5 and group 1 are still significantly negative.

The sample period from January 1996 to June 2005 contains the inflation and deflation of the stock market bubble as well as a more normal time following the deflation of the bubble. Ofek, Richardson and Whitelaw (2004) and Battalio and Schultz (2006) examine the connection between the bubble and option prices. Hence, it is natural to ask to what extent the results are driven by the bubble. Panels C and D of Table VI contain the results of re-doing the analysis on the subperiods from January 1996 through July 2001 and August 2001 through June 2005. The former contains the bubble (inflation and deflation) and the latter is post-bubble. Both the point estimates and test statistics for the group 5 returns and the difference between the group 5 and group 1 returns are similar in both subperiods (and similar to those from the entire period.) It does not appear that the paper's findings are driven by the stock market bubble. It is also noteworthy that overall the stock market rose during both subperiods. As explained above, a rising stock market biases against finding both anomalies.

Up to this point the analysis has been done using bid-ask midpoints. Sometimes there is a concern when research shows anomalous results using bid-ask midpoints, because midpoint prices generally are better than those at which an investor can buy or sell securities. This concern does not apply in the present setting. I use the bid-ask midpoint, because it is the best proxy for the true price of the call options. (This is the reason, for example, that sophisticated practitioners and researchers imply volatilities from option bid-ask midpoints.) It is true that investors generally could not buy the call at the midpoint price. Instead, they would have to pay the higher ask price, or perhaps a price somewhere in between the bid-ask midpoint and the ask price. However, using higher call prices will just exacerbate the anomalies, since it will decrease call returns in general and OTM calls disproportionately. In other words, in the present context the use of bid-ask midpoints makes the test more conservative.

Nonetheless, I have re-computed the results using call ask prices and the results are presented in Panel E of Table VI. As expected, the anomalies are larger in both magnitude and statistical significance. For completeness, I also include the results computed from bid prices in Panel F of Table VI. Here the point estimates still violate the call return restrictions, and the results are weaker because the bid ask spread costs increase as call prices decrease. They are not especially relevant though as the bid price is clearly an inferior proxy (as compared to the bid-ask midpoint) for the prices that investors would pay for the calls.

I also investigate the returns to held to maturity calls that have 2 months to expiration. Panels G report those call returns for different strike groups. The return of strike group 5, and the return difference between group 5 and group 1 are still negative, but with smaller magnitude and less statistical significance compared with 1 month calls. When strike groups are classified by K/S and Black Scholes delta, the returns in group 5 are less than –17% with t-statistics less than –2.12. When strike groups are classified by ln(K/S)/σ, the negative returns in strike group 5 are marginally significant.

V. Potential Explanations for the Puzzling Call Returns

This section of the paper investigates potential explanations for the puzzling call returns. In the first subsection, I present evidence that negative call returns are due to excessively high call prices rathern than unusually low stock returns; in the second subsection, I explore why the call prices are so high. Negative call returns can be driven by (1) low stock returns, (2) high call prices, or a combination of.(1) and (2). The 'low' stock return explanation for the negative OTM calls returns is that the realized returns to the underlying stocks during the sample period had a smaller than representative number of large positive returns and hence the OTM calls returns were unusually small. This explanation would be a version of the 'peso' problem discussed, for example, by Jackwerth and Rubinstein (1996), Ait-Sahalia, Wang, and Yared (2001) and Broadie, Chernov, and Johannes (2007). The second potential explanation is that the market prices of OTM calls are too high so that the call returns are too low even if the realized underlying stock returns are representative of the distribution from which they are drawn. It will be seen in subsection A that the evidence supports the second explanation of high call prices but not the first explanation of low stock returns. In the subsection B, I explore the reason for the high call prices focussing on the possibility that investor utility functions are not globally concave, because global concavity is the one of the two assumptions for the call return restrictions. Global concavity will be violated by investors seeking either idiosyncratic skewness or idiosyncratic volatility. Such risk-seeking will increase call prices and, in principle, can make their returns violate the Coval and Shumway restrictions.

A. Low Stock Returns or High Call Prices?

In a short sample period, it would not be unusual for stocks to experience returns that are low relative to their true distribution. If this is the case, even if market call prices are set so that their expected returns are positive and increasing in strike price, the realized, average call returns may be negative or decreasing in strike price. In this subsection, I investigate this possibility using three different methods. The first method estimates call prices independently from the underlying stock price movements, and then compares the estimated call prices with the market call prices. The results show that the prices of ATM and OTM calls observed in the market are higher than the estimated call prices. The second method replaces observed market call prices with those calculated from the Black-Sholes (1972) model or the Merton (1976) jump model with or without jump premia or adjusted for 'peso' problem. These replacements all result in call returns that are positive and increasing in strike prices, consistent with the Coval and Sumway (2001) restrictions. This finding indicates that the negative call returns computed from market call prices do not result from low realized stock returns but rather from high call market prices, because the actual realized stock returns would have resulted in positive call returns had the option prices conformed to standard option prices models. The third method compares the frequency of large monthly stock returns during the sample period of this study, which is from 1996 to 2005, with the frequency of large monthly stock returns from 1963 to 1995. This analysis shows that from 1996 to 2005 the frequency of large monthly stock returns, especially for optionable stocks, is higher than from 1963 to 1995. This fact constitutes straightforward evidence that the quantity of large returns from 1996 to 2005 was not low.

A.1 Market Call Prices and Estimated Call Prices

Relative to a model, we can determine ex-post whether a security is mispriced regardless of length of time period. For example, if the market prices of a stock at times t and $t+1$ are P_t and P_{t+1}, suppose r_t^M, the market return from t to $t+1$, is the only factor that affects the stock return, and the risk free rate is zero, then the estimated stock price at *t*, $\widehat{P}_t$, is

$$\widehat{P}_t = (1+\beta r_t^M)^{-1} P_{t+1}.$$

The estimated price $\widehat{P}_t$ is independent of market factor because $\widehat{P}_t$ is generated by discounting P_{t+1} with $(1+\beta r_t^M)$, while P_{t+1} is driven by $(1+\beta r_t^M)$. By comparing P_t with $\widehat{P}_t$ we can judge whether P_t is over- or under-valued, regardless of the length of sample period or the magnitude of market return. For example, in a short time period when market experiences negative returns, after discounting P_{t+1} by the negative market factor, we can still have the correct $\widehat{P}_t$ which is higher than P_{t+1}. This implies that the negative stock return is not because a high P_t but because a low r_t^M.

To conduct this type of exercise in the option market, it is reasonable to start with the Black Scholes model. Let *t* denote the current date, $T \geq t$ the option expiration date, and S_t the stock price at date t. Assume the stock does not pay dividends and S_t follows geometric Brownian motion. Let V be the option pricing function given by the Black-Scholes formula, so that V_t is the option price at time t, and V_T is the option price at time T. Under these assumptions, I show in the Appendix that the call estimated price at date t is

$$V_t = \exp\left\{-\int_t^T \Omega_u \frac{dS_u}{S_u} - \int_t^T \left[\left(-r\Omega_u + r\right) - \frac{1}{2}\Omega_u^2\sigma^2\right] du\right\} V_T, \tag{11}$$

$$\text{s.t. } V_T > 0,$$

where $\Omega_t \equiv \frac{S_t}{V_t}\frac{\partial V}{\partial S}$ is the call price elasticity with respect to stock price. In words, the option estimated price V_t can be considered as V_T discounted by the realized return on the option, $\int_t^T \frac{\Omega_u}{S_u} dS_u + \int_t^T \left[\left(-r\Omega_u + r\right) - \frac{1}{2}\Omega_u^2\sigma^2\right] du$. Here both V_T and the realized option return are determined by realized stock return path from time t to T; if stock return is low, both V_T and option realized return will be small, if stock return is high, then both V_T and option realized return will be high. Therefore the estimated option price V_t takes out any effect of stock price movements during the sample period and thus is not subject to the concern that large returns are under-represented in a short sample period.

In practice, I estimate the integral in equation (11) by a discrete (daily) sum where σ is the realized volatility from date t to T. If the call final payoff is above zero, V_T is the final payoff to the call. Since equation (11) is not valid when V_T is zero, when the final call payoff is zero, I estimate V_T using the Black-Scholes formula assuming the call has one half day to maturity, and the stock price is the closing stock price on expiration Friday.

To compare V_t with the market call price, I calculate the relative price of strike group G in following way:

$$RP_t^G = \frac{1}{N}\sum_i^N \left(\frac{V_{t,i}}{C_{t,i}} - 1 \right) \tag{12}$$

where $C_{t,i}$ is the market call price, and $V_{t,i}$ is estimated using equation (11). If the call market price is higher than estimated call price under Black-Scholes, RP_t^G will be less than zero.

Panel A of Table VII reports the relative call prices for one month options of five strike groups, where the market call price is the mid point of the bid and ask prices. In strike group 1 the relative prices are close to zero. For strike group 2 to strike group 5, the relative prices are all significantly negative. Furthermore, the relative prices tend to be more negative for higher strike groups. For example, when *K/S* is used to classify strike groups, the relative price is –0.07 for group 2, and -0.39 for group 5.

Panel B of Table VII reports the relative price calculated from call ask prices. Here the relative prices are more negative than those computed from midpoints of bid and ask prices and also decreasing in the strike group. Turning to the results using call bid prices in Panel C of Table VII, we see that in strike group 1, the relative prices are positive. For groups 2 to 5, the relative prices are all negative but with less significant *t*-tests.. If the call prices were correct, the bid relative prices would be positive due to the bid ask spread. The negative bid relative price implies that even the bid prices are too high relative to the estimated model prices.

The key assumption in generating the relative prices is that Black-Scholes is the correct model to price call options that do not pay dividends. To the extent that the Black-Scholes model does not hold, then equation (11) measures not only mispricing but also model misspecification such as that due to jump and stochastic volatility components.

A.2. Call Returns with Theoretical Call Prices

This section further investigates the source of the negative call returns by examining the one month call returns computed from Equation (8) when the observed call market prices are replaced by theoretical call prices under the Black-Scholes model or the Merton (1976) jump model without jump premia, with jumps adjusted for the 'peso' problem, and with jump premia. If negative call returns from observed market prices have their source in call prices that are too high rather than a 'peso' problem or jumps, then the call returns calculated from the theoretical prices will be positive and increasing in strike price. On the other hand, if the negative call returns are driven by a 'peso' problem and/or jumps, the call returns calculated from theoretical prices will still be negative.

In implementing the Merton (1976) jump price model, I estimate each stock's physical jump parameters from available daily returns from January 1996 to June 2005. A daily return is classified as a jump if it is at least 3 standard deviations from the stock's daily mean return during the sample period. The physical jump probability λ^P is the total number of jumps divided by number of years when returns are available, and the physical average jump size κ^P is the average of those jump returns. The stock return volatilities are estimated using the previous 60 days of daily returns for both the Black-Scholes and Merton jump models. Models with stochastic volatility are not considered, because my analysis uses one month options and volatility tends to cluster over short time periods.

Panels A and B of Table VIII report the call returns under the Black-Scholes model and the Merton jump model without jump premia, where the jump size and jump probability are the physical jump size and physical jump probability. The call returns are positive and increasing in strike price in both Panels A and B. For example, for the *K/S* strike group 5 the average call returns are 63% (*t*-test = 2.38) under Black-Scholes, and 47% (*t*-test = 2.11) under the Merton jump model. Panels A and B also show that call returns calculated from the Merton jump model are closer to zero than those computed from Black-Scholes. These results are consistent with the simulations in Broadie, Chernov, and Johannes (2007), and imply that when jumps are priced without risk premia, option returns tend toward zero.

Panel C reports the call returns adjusted for the 'peso' problem. Following Broadie, Chernov, and Johannes (2007), I consider a jump probability λ^Q three times the physical jump probability λ^P, and keep the jump size κ^Q at the same level as the physical jump size κ^P. The effect on call returns after changing the jump probability depends on the sign of the jump size. If κ^P is positive, increasing the jump probability increases the call price, and, hence, reduces the call return. If κ^P is negative, increasing the jump probability will increase call returns. Comparing Panel C with Panel B, we can see that except for *K/S* group 5, all other call returns in Panel C are smaller than those in Panel B, reflecting the fact that the average jump size are positive for individual stocks.[4]

We can also see from Panel C that even though the embedded jump probabilities in call prices are two times higher than the physical probabilities, the call returns are still positive and increasing in strike price with economic and statistical significance. For example, for the log(K/S)/σ group 5, the one month held to expiration return is 71.50% (*t-stat* = 4.83), and the return difference between group 5 and group 1 is 71.98 (*t-stat* =5.06). These results provide evidence that it is unlikely that a 'peso' problem produces the puzzling negative call returns computed from market prices.

Panel D reports the call returns when the call prices include jumps and jump premia. Here the jump probability λ^Q is set to twice the physical jump probability λ^P, and the jump size κ^Q is 0.04 less than the physical jump size κ^P. These jump premia are similar to those used in Liu, Pan, and Wang (2005), and changing them to other reasonable values does not affect the general pattern of returns. We see from Panel D that the call returns are positive and increasing in strike price when the jump risk premia is priced in call options. Note also that the call returns in Panel D are larger than those in Panel C, because reducing the jump size decreases the call prices. These results in Table VIII indicate that the negative OTM call returns are not due to low stock returns or jumps but rather are rooted in high call prices.

A.3 Historical probability of large returns

In this subsection I further investigate the 'peso' problem by comparing the frequency of large returns from 1996-2005 to the frequency prior to 1996. If large stock returns are more frequent over 1996-2005 than before 1996, then it is unlikely that my findings are driven by fewer large stock returns than investors expected over the paper's data period.

Table IX presents the frequencies of large monthly returns for the two time periods, 199601 to 200506 and 196301 to 199512. The frequency of monthly returns larger than 15% (or 20% or 30%) for a particular set of stocks is the number of returns larger than 15% (or 20% or 30%) during the time period divided by total number of monthly stock returns during the time period.

Panel A of Table IX shows that for all stocks the frequencies of large returns are higher during 1996-2005 than during 1963-1995. For example, 12.23% of stock-months during 1996-2005 have monthly returns larger than 15%, and only 10.98% during 1963-1995.

Panel B of Table IX reports the probability of large returns for optionable stocks. Here I define a stock to be optionable in month t if it has at least one call in calculating the one month call returns reported in Table V. Mayhew, Stewart, and Mihov (2004) show that exchanges are more likely to select stocks for option listing if the stocks have large size, volume, or volatility. Consequently, before 1996 I select stocks based on their sizes, volumes, or volatilities such that the selected stocks have the same size, volume, or volatility deciles as the optionable stocks during 1996-2005. In particular, I first count the number of optionable stocks belonging to each decile for every month in 199601-200506. I then generate the average number of stocks in each decile across the 113 months, and randomly select with replacement the same number of stocks in each decile in every month during 1963-1996 to get a set of matched stocks.

Panel B of Table IX shows that the frequencies of large returns for optionable stocks are also higher than the frequencies for matched stocks before 1996. Indeed, 13.91% of optionable stocks have monthly returns higher than 0.15, and 8.98% and 4.13% of optionable stocks have monthly returns higher than 0.20 and 0.30 respectively. On the other hand, the average frequencies for having returns above 0.15 (or 0.20 or 0.30) for matched stocks during 1963-1995 are only 10.61% (or 6.49% or 2.91%).

Not every optionable stock has call options in the highest strike group 5 every month. Panel C of Table IX reports the frequencies of large returns for optionable stocks having calls in strike group 5. The optionable stocks in strike group 5 have the highest likelihood of large returns; for example on average 18% of stocks have monthly returns above 0.15. This probability is not only higher than that for matched stocks before 1996, but is also higher than that for all optionable stocks.

If the period from 1963 through 2005 is long enough for rare large individual stock returns to be realized, Table IX implies these extreme large returns are more likely to be realized during the period from 199601 through 200506 than the period before 1996. This analysis makes it unlikely that a 'peso' problem is the reason for the negative OTM call returns.

B. Why are call prices high?

The previous subsection argues that the negative call returns are driven by high call prices. In this subsection I explore why the call prices are so high by focusing on one of the assumptions that is made to derive the call return restrictions. The call return restrictions are derived from the assumptions that (1) investors have increasing and concave utility functions and (2) stock returns are positively correlated with aggregate wealth. It seems unlikely that assumption (2) would be violated by more than a very small number of stocks. Hence, I focus on the possibility that assumption (1)'s supposition that investor utility functions are everywhere concave is violated.

B.1 Idiosyncratic skewness and volatility

The great majority of modern financial theory assumes that investors are uniformly risk-averse (i.e., have uniformly concave utility functions). This view, however, has been far from unanimous. In fact, a number of the greatest financial economists of all time have maintained that risk seeking and risk aversion co-exist within investors.

For example, in *The Wealth of Nations*, Adam Smith (1776) writes:

> The ordinary rate of profits always rises more or less with the risk. It does not, however, seems to rise in proportion to it, or so as to compensate it completely…. The presumptuous hope of success seems to act here as upon all other occasions, and to entice so many adventurers into those hazardous trades, that their competition reduces the profit below what is sufficient to compensate the risk.

More recently, Friedman and Savage (1948) and Markowitz (1952) also noted that risk aversion and risk seeking both play a role in people's behavior. They modified the standard utility model to rationalize people buying high-priced lotteries. In a similar vein, Kahneman and Tversky (1979, 1992) suggest that people place decision weights on the probabilities of each outcome and tend to overweight the small probabilities of extreme events.

Barberis and Huang (2005) analyze the cumulative probability weighting of prospect theory which incorporates both risk aversion and risk seeking. They present a model in which people overweight small probabilities of extreme events and securities with high enough idiosyncratic skew are over-priced. Their theory also explains why people are willing to pay more than the expected payoff for risky lotteries. Mitton and Vorkink (2005) reach similar conclusions within a model where agents have heterogeneous preferences for skewness. Brunnermeier and Gollier (2007) also show that if people are optimistic about the payout from their investment positively skewed assets tend to have low returns. Kumar (2005) suggests that stocks with low institutional ownership have a negative idiosyncratic skewness premium.

As we have seen above, stock call returns are highly skewed and their skewness increases with strike price. Over-weighting the small probabilities of large stock price movements has the potential to explain the negative OTM calls returns and call returns decreasing with strike price.[5]

If both buyers and sellers overweight the small probability of large stock movements, then buyers are willing to buy OTM calls at high prices, and sellers are willing to sell OTM calls only when the prices are high. In equilibrium, the OTM call prices will be higher than under standard risk aversion.

Ang et al (2006) find that over the 1963 to 2000 period high idiosyncratic volatility stocks earn lower returns than low idiosyncratic volatility stocks. Using earlier data, Pratt (1966) and McEnally (1974) similarly find that the average return to high volatility stocks is less than the average return on moderate volatility stocks. It is possible that these stock return patterns have their source in investors preferring idiosyncratic volatility which would imply that they do not have globally concave utility functions. Since calls magnify the idiosyncratic volatility of stocks, and the magnification is greater for more OTM calls, investors seeking idiosyncratic volatility is another potential explanation for the call return restriction violations.

B.2 Evidence on Idiosyncratic Skewness and volatility

I begin investigating whether idiosyncratic skewness and/or volatilty may contribute to the call return restriction violations by examining the expected and realized idiosyncratic skewness and volatility of call returns for different strike groups. I estimate the expected idiosyncratic skewness and volatility in a strike group by averaging the idiosyncratic skewness and volatility of the calls in the strike group based on the distribution of the previous 30 monthly returns on the underlying stock. In particular, I assume that over the next month the stock return is drawn from the historical distribution and then calculate the return to holding for one month to maturity. The expected idiosyncratic skewnewss and volatility are calculated from these call returns. The realized idosyncratic skewness and volatility are the average of the time-series of 113 monthly skewness and volatility of realized call returns in each strike group.

Panels A and B of Table X report, respectively, the average expected and realized idiosyncratic skewness and volatility for the five strike groups. Both expected and realized idiosyncratic skewness and volatility are monotonically increasing in the strike group. For strike group 1, the expected idiosyncratic skewness is approximately 0.40, while for strike group 5 it is about 3.50. The realized idiosyncratic volatility is also increasing in strike price and more dramatically than the expected idiosyncratic volatility. The realized idiosyncratic skewness is around 0.80 for group 1 and above 7 for group 5. The monthly expected idiosyncratic volatility is about 50% for group 1, and close to 400% for group 5. The increasing idiosyncratic skewness and volatility across strike groups are consistent with both idiosyncratic skewness and volatility playing a role in producing the call return restriction violations.

Since idiosyncratic volatility and skewness are positively correlated, it is possible that after controlling for one the other is not important for call option returns. In order to investigate this possibility, I sort calls independently into quintiles of expected idiosyncratic skewness and volatility and calculate the call returns in each of the 25 resulting categories. If both idiosyncratic skewness and volatility have the potential to explain call returns, the call portfolio returns will be decreasing both in idiosyncratic skewness and in indiosyncratic volatility.

Table XI reports the average call returns for the 25 idiosyncratic skewness and volatility categories. The call returns are decreasing in idiosyncratic skewness for all five idiosyncratic volatility quintiles: the average return in the lowest idiosyncratic skewness quintile (across the five idiosyncratic volatility quintiles) is 5.74%, while the average return in the highest idiosyncratic skewness quintile (again across the five idiosyncratic volatility quintiles) is –21.53%. Panel B of Table XI reports that the differences between the highest and lowest idiosyncratic skewness quintiles are statistically significant for all idiosyncratic volatility quintiles. For idiosyncratic volatilities, however, call returns are no longer decreasing in volatility after controlling for idiosyncratic skewness. On the contrary, call returns are increasing with idiosyncratic volatility for four out of five of the skewness quintiles. These results are consistent with part of the puzzling call returns coming from investors seeking idiosyncratic skewness. It appears that seeking of idiosyncratic volatility is less likely to play a role.

VI. Conclusion

This paper finds that stock call returns exhibit patterns that conflict with predictions derived from very weak economic assumptions. In particular, I find that average OTM call returns are negative and that call returns are decreasing in strike price. From January 1996 through June 2005, a period when stock prices rose overall, calls with K/S ratio greater than 1.15 experienced average one month returns of −36.86% and calls with K/S ratio greater than 1.15 experienced average returns that were 34.86% lower than calls with K/S ratio less than 0.85. These puzzling findings are robust to different methods for assigning calls to moneyness categories, to limiting the analysis to calls that actually traded, to splitting the sample into the subperiod where the stock market bubble inflated and deflated and the post-bubble period, and to controlling for realized stock price paths. Evidence is also presented that the negative returns are not driven by a 'peso' problem or jump risk premia.

The paper also takes a first step toward understanding the puzzling call option returns. Results are provided that are consistent with the puzzle having its source in investors sometimes being risk-seeking. Future work should further explore this possibility and consider others as well.

Appendix: Proof of equation (11)

Let t denote the current date, τ an option's time to maturity, and S_t the stock price at date t. Assume the stock does not pay a dividend, S_t follows geometric Brownian motion, and the Black-Scholes formula holds. Let V_t be the Black-Scholes option price at time t.

From Ito's lemma, the dynamics of the option price are

$$dV_t = \frac{\partial V}{\partial S} dS_t + \frac{1}{2}\frac{\partial^2 V}{\partial S^2}\sigma^2 S_t^2 dt + \frac{\partial V}{\partial t} dt \ . \tag{A1}$$

The standard option pricing p.d.e. is

$$\frac{1}{2}\frac{\partial^2 V}{\partial S^2}\sigma^2 S^2 + \frac{\partial V}{\partial t} = -\frac{\partial V}{\partial S} rS + rV \ . \tag{A2}$$

Substituting (A2) into (A1), the option price dynamics become

$$\begin{aligned} dV_t &= \frac{\partial V}{\partial S} dS_t - \frac{\partial V}{\partial S} rS_t dt + rV_t dt \\ &= V_t \left[\Omega_t \left(\frac{dS_t}{S_t} - rdt \right) + rdt \right] \end{aligned} , \tag{A3}$$

where the option omega is defined as $\Omega_t \equiv \frac{S_t}{V_t}\frac{\partial V}{\partial S}$.

Based on Itô's lemma and equation (A1) above, the dynamics of $\ln V_t$ are

$$\begin{aligned} d\ln V_t &= \frac{1}{V_t} dV_t - \frac{1}{2}\frac{1}{V_t^2}(dV_t)^2 \\ &= \frac{1}{V_t} dV_t - \frac{1}{2}\frac{1}{V_t^2}\left(\frac{\partial V_t}{\partial S_t}\right)^2 \sigma^2 S_t^2 dt \qquad \text{s.t } V_t > 0 \end{aligned} \ . \tag{A4}$$

Then using equation (A3), equation (A4) becomes:

$$d\ln V_t = \left(\Omega_t\left(\frac{dS_t}{S_t} - rdt\right) + rdt\right) - \frac{1}{2}\frac{1}{V_t^2}\left(\frac{\partial V}{\partial S}\right)^2 \sigma^2 S_t^2 dt\,. \tag{A5}$$

Let V_T be the price of stock option at time T, then $\frac{V_T}{V_t}$ can be expressed as

$$\begin{aligned}
\frac{V_T}{V_t} &= \exp\left[\ln V_T - \ln V_t\right] = \exp\left[\int_t^T d\ln V_u du\right] \\
&= \exp\left\{\int_t^T \Omega_u \frac{dS_u}{S_u} + \int_t^T \left[\left(-r\Omega_u + r\right) - \frac{1}{2}\frac{1}{V^2}\left(\frac{\partial V}{\partial S}\right)^2 \sigma^2 S_u^2\right] du\right\}, \\
&= \exp\left\{\int_u^T \Omega_u \frac{dS_u}{S_u} + \int_u^T \left[\left(-r\Omega_u + r\right) - \frac{1}{2}\Omega_u^2 \sigma^2\right] du\right\}
\end{aligned} \tag{A6}$$

s. t. V_{t} , $V_u > 0$ and $V_T > 0$.

Then after rearranging (A6), we have

$$V_t = \exp\left\{-\int_t^T \Omega_u \frac{dS_u}{S_u} - \int_t^T \left[\left(-r\Omega_u + r\right) - \frac{1}{2}\Omega_u^2 \sigma^2\right] du\right\} V_T \qquad \text{s. t. } V_T > 0 \tag{A7}$$

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Table I
Summary of Stock Option Trading, 1996 – 2004

Option volume and option open interest are the sum of total daily stock option volume and open interest. Stock market volume is the sum of total daily stock share volume.

Year	Stock Option Volume (000,000)	Stock Option OpenInt (000,000)	Stock Market Volume (000,000)
1996	158	2,947	267,745
1997	220	4,123	327,375
1998	269	5,369	407,482
1999	366	7,415	520,256
2000	555	10,937	768,369
2001	600	13,195	856,067
2002	601	14,796	913,258
2003	683	18,013	911,830
2004	838	18,095	941,316

Table II
Characteristics of Underlying Stocks on Option Expiration Dates with Various Numbers of One Month to Expiration Calls

This table reports underlying stock characteristics measured on each expiration date as a function of the number of one month to expiration calls. (i.e., Ks). ‘No of Ks’ (strike prices) is defined as the number of strike prices for an underlying stock for one-month calls on the option expiration day of each month. Options are deleted if (1) the underlying stock has an ex-dividend date during the remaining life of its one month options, (2) the option bid price is less than or equal to $0.125, or (3) the call price violates a no-arbitrage bound. Number of stocks is the average number of stocks across the 113 months from January 1996 through May 2005. Size is the average log of end-of-month shares outstanding multiplied by stock price. Stock price is the average stock price on the option expiration date of each month. Market beta is the slope coefficient from the regression of the previous 24 monthly stock returns on market returns. Volatility is the annualized sample estimate from the previous 60 daily stock returns. The time period is from January 1996 through June 2005.

No of Ks	No of Stocks	Log(Size)	Stock Price	Mkt Beta	Volatility
1	401.97	13.12	16.06	0.87	0.50
2	506.17	13.60	23.63	0.93	0.48
3	389.69	13.88	30.57	1.05	0.50
4	211.35	14.19	36.34	1.21	0.55
5 and above	268.06	14.77	49.79	1.42	0.57

Table III
Strike Groups

This table defines five strike group ranges for K/S, log(K/S)/σ, and Delta. Strike group 1 contains calls with the lowest strike prices, and group 5 contains calls with the highest strike prices. K/S is the strike price divided by the closing price. σ is the annualized stock volatility computed from the previous 60 days of daily returns. Delta is the Black Scholes (1973) delta where volatility is the annualized stock volatility from the previous 60 days of daily returns.

Strike Group	1	2	3	4	5
R=K/S	R≤0.85	0.85<R≤0.95	0.95<R≤1.05	1.05<R≤1.15	1.15<R
R=log(K/S)/σ	R≤-0.3	-0.3<R≤-0.1	-0.1<R≤0.1	0.1<R≤0.3	0.3<R
R = Delta	0.85≤R≤1	0.65≤R<0.85	0.35≤R<0.65	0.15≤R<0.35	0≤R<0.15

Table IV
Summary Statistics for Call Options and Underlying Stocks

This table reports summary statistics for returns to one month call options held to maturity for low to high strike groups and their underlying stocks. The reported summary statistics are monthly averages. We assign each option to a strike group based on its value of K/S, log(K/S)/σ, or BLS-delta. Table III defines the range of K/S, log(K/S)/σ, or BLS-delta for each strike group. Strike Group 1 is the lowest strike price, and strike group 5 is the highest strike price. The time period is from January 1996 through June 2005.

Strike Group	1	2	3	4	5
Panel A: K/S					
Call Beta	3.51	6.19	10.82	14.07	14.23
Call Volume (Contract)	5.65	34.15	148.38	145.34	115.87
Call Volume (Dollar)	11,882	31,042	40,509	19,386	8,247
Stock Beta	1.21	1.10	1.11	1.25	1.64
Stock Returns	0.83	0.87	0.65	1.29	0.39
Call Portfolio Return Stand Deviation	0.24	0.37	0.56	0.71	0.81
Call Portfolio Return Autocorrelation	0.03	-0.01	-0.03	0.03	0.08
Call Portfolio Return Skewness	-0.17	0.00	0.45	1.23	2.44
No. of Obs	1,293.35	1,238.66	1,235.62	864.94	458.12
Panel B: log(K/S)/σ					
Call Beta	3.78	7.57	10.93	14.67	12.02
Call Volume (Contract)	5.65	59.78	167.65	144.66	101.15
Call Volume (Dollar)	12,269	32,928	42,656	16,083	5,278
Stock Beta	1.06	1.18	1.22	1.23	1.21
Stock Returns	0.84	0.92	0.71	1.07	1.11
Call Portfolio Return Stand Deviation	0.24	0.41	0.58	0.72	0.85
Call Portfolio Return Autocorrelation	0.01	-0.01	-0.02	0.02	0.17
Call Portfolio Return Skewness	-0.42	0.09	0.56	1.23	2.91
No. of Stocks	1,402.34	1,049.18	1,446.57	484.53	359.59
Panel C: BLS_delta					
Call Beta	4.12	8.58	11.61	15.04	11.67
Call Volume (Contract)	6.91	93.13	182.29	139.70	101.74
Call Volume (Dollar)	13,043	35,292	39,560	12,216	5,039
Stock Beta	1.09	1.22	1.23	1.22	1.18
Stock Returns	0.86	1.09	0.62	1.06	1.04
Call Portfolio Return Stand Deviation	0.27	0.46	0.60	0.73	0.93
Call Portfolio Return Autocorrelation	0.00	-0.01	-0.01	0.03	0.16
Call Portfolio Return Skewness	-0.31	0.22	0.63	1.33	4.09
No. of Stocks	1,572.34	870.28	1,084.26	803.87	448.77

Table V
Returns to Holding One Month Call Options to Expiration

This table reports averages of monthly time series of the average return to holding one month call portfolios to expiration for various strike groups and differences in strike groups. Each call is assigned to a strike group based on its value of K/S, log(K/S)/σ, or Black-Scholes delta (BLS_delta), where σ is the annualized stock return volatility estimated from the previous 60 daily returns. This is also the volatility used when computing Black-Scholes deltas. Table III defines the range of K/S, log(K/S)/σ, or BLS_delta for each strike group. Strike Group 1 refers to the lowest strike prices, and group 5 contains the highest strike prices. *X*'-*Y*' is the call return difference between strike group *X* and group *Y* for the same underlying stocks. Call returns are computed from bid-ask midpoints, and each month the average call returns on various underlying stocks are averaged to obtain the month's call return for the group. The t-stat is the average call portfolio return divided by the standard error of the time series monthly average call portfolio returns. Call options are selected on each option expiration date that mature the next expiration date if all the following conditions are satisfied: (1) the underlying stock does not have ex-dividend date during the remaining life of the option, (2) the bid price is strictly larger than $0.125, and (3) the call price satisfies a no-arbitrage restriction. The time period is from January 1996 through June 2005.

Strike Group	1	2	3	4	5	5' - 1'	4' - 1'	5' - 2'
K/S (%)	2.22	2.68	1.98	-8.05	-27.84	-27.90	-12.17	-22.38
t-Stat	(0.90)	(0.73)	(0.38)	(-1.21)	(-3.70)	(-5.02)	(-2.72)	(-4.38)
log(K/S)/σ (%)	3.05	2.18	0.54	-8.70	-26.98	-29.32	-11.92	-27.50
t-Stat	(1.25)	(0.54)	(0.10)	(-1.29)	(-3.79)	(-5.09)	(-2.61)	(-5.02)
BLS_delta (%)	2.96	1.84	-0.32	-10.45	-29.78	-31.18	-13.79	-27.69
t-Stat	(1.18)	(0.44)	(-0.06)	(-1.50)	(-3.87)	(-4.72)	(-2.87)	(-3.48)

Table VI

Returns to Holding One Month Call Options to Expiration, Robustness Tests

This table reports averages of monthly time series of the average return to holding one month call portfolios to expiration for various strike groups and differences in strike groups. Each call is assigned to a strike group based on its value of K/S, log(K/S)/σ, or Black-Scholes delta (BLS_delta), where σ is the annualized stock return volatility estimated from the previous 60 daily returns. This is also the volatilty used when computing Black-Scholes deltas. Table III lists the range of K/S, log(K/S)/σ, or BLS_delta for each strike group. Strike Group 1 refers to the lowest strike prices, and group 5 contains the highest strike prices. *X'*-*Y'* is the call return difference between strike group *X* and group *Y* for the same underlying stocks. Call returns are computed from bid-ask midpoints except Panel E and F, and each month the average call returns on various underlying stocks are averaged to obtain the month's call return for the group. Call options are selected on each option expiration date that mature the next expiration date if all the following conditions are satisfied: (1) the underlying stock does not have ex-dividend date during remaining life of the option, (2) the bid price is strictly larger than $0.125, and (3) the call price satisfies a no-arbitrage restriction. Panel A restricts the analysis to call options that are traded, Panel B to calls on the largest 200 market capitalization stocks, Panel C to the stock market bubble period, and Panel D to the post-bubble period. Panel E computes call returns from ask prices, Panel F computes the call returns from bid prices. Panel G reports two and three month call returns. The time period is from January 1996 through June 2005.

Strike Group	1	2	3	4	5	5' - 1'	4' - 1'	5' - 2'
Panel A Traded Options								
K/S (%)	0.69	1.19	1.31	-8.45	-22.84	-18.31	-9.82	-14.30
t-Stat	(0.22)	(0.29)	(0.24)	(-1.21)	(-2.61)	(-2.41)	(-1.84)	(-2.04)
log(K/S)/σ (%)	2.61	1.30	-0.29	-9.26	-24.64	-19.01	-11.44	-23.68
t-Stat	(0.91)	(0.30)	(-0.05)	(-1.30)	(-2.93)	(-1.83)	(-2.19)	(-2.95)
BLS_delta (%)	2.44	1.05	-1.36	-10.83	-27.30	-19.55	-13.18	-25.15
t-Stat	(0.82)	(0.23)	(-0.23)	(-1.46)	(-2.59)	(-1.36)	(-2.27)	(-1.91)
Panel B: Largest 200 stocks								
K/S (%)	5.90	4.73	4.67	-7.19	-26.73	-28.94	-13.82	-24.51
t-Stat	(2.73)	(1.36)	(0.80)	(-0.87)	(-2.29)	(-2.53)	(-1.96)	(-2.34)
log(K/S)/σ (%)	4.11	5.13	3.45	-4.64	-24.31	-27.22	-9.14	-30.60
t-Stat	(1.74)	(1.22)	(0.59)	(-0.58)	(-2.51)	(-3.10)	(-1.47)	(-3.63)
BLS_delta (%)	4.24	5.28	2.61	-5.24	-26.93	-28.62	-9.43	-34.20
t-Stat	(1.75)	(1.20)	(0.43)	(-0.62)	(-2.59)	(-2.98)	(-1.43)	(-3.75)
Panel C: 199601- 200205								
K/S (%)	1.19	2.00	1.33	-9.30	-29.70	-28.78	-12.24	-25.03
t-Stat	(0.40)	(0.45)	(0.20)	(-1.12)	(-2.94)	(-3.37)	(-2.04)	(-3.86)
log(K/S)/σ (%)	2.15	1.51	0.03	-8.86	-26.41	-28.20	-10.83	-25.52
t-Stat	(0.71)	(0.30)	(0.00)	(-1.04)	(-2.91)	(-3.87)	(-1.94)	(-3.72)
BLS_delta (%)	2.10	1.06	-1.07	-10.67	-31.60	-31.92	-12.99	-29.89
t-Stat	(0.66)	(0.20)	(-0.15)	(-1.20)	(-3.51)	(-4.24)	(-2.18)	(-4.08)

Table VI – *Continued*

Panel D: 200206-200506								
K/S (%)	2.25	2.47	1.40	-7.39	-23.96	-24.82	-11.56	-19.59
t-Stat	(0.70)	(0.53)	(0.22)	(-0.90)	(-2.51)	(-3.63)	(-2.20)	(-3.13)
log(K/S)/σ (%)	2.72	2.83	1.24	-8.48	-27.81	-28.45	-11.65	-29.98
t-Stat	(0.68)	(0.42)	(0.14)	(-0.76)	(-2.40)	(-3.00)	(-1.49)	(-3.30)
BLS_delta (%)	2.69	2.71	0.74	-10.13	-27.13	-27.66	-13.09	-24.21
t-Stat	(0.66)	(0.39)	(0.08)	(-0.89)	(-1.97)	(-2.29)	(-1.62)	(-1.47)
Panel E: Ask Price								
K/S (%)	-1.37	-2.19	-6.30	-20.22	-39.10	-35.97	-20.46	-29.78
t-Stat	(-0.58)	(-0.63)	(-1.31)	-(3.51)	(-6.12)	(-7.84)	(-5.43)	(-7.07)
log(K/S)/σ (%)	-0.71	-2.99	-7.39	-20.90	-39.51	-39.03	-20.75	-35.80
t-Stat	(-0.30)	(-0.78)	(-1.48)	-(3.56)	(-6.62)	(-8.29)	(-5.43)	(-7.87)
BLS_delta (%)	-0.89	-3.51	-9.00	-23.26	-41.96	-40.78	-23.22	-36.04
t-Stat	(-0.37)	(-0.89)	(-1.74)	(-3.88)	(-6.49)	(-7.48)	(-5.86)	(-5.41)
Panel F: Bid Price								
K/S (%)	6.40	8.50	13.44	12.06	-8.35	-12.94	2.63	-7.84
t-Stat	(2.49)	(2.18)	(2.28)	(1.48)	(-0.88)	(-1.77)	(0.46)	(-1.16)
log(K/S)/σ (%)	7.43	8.43	11.43	11.31	-4.75	-10.76	3.95	-11.07
t-Stat	(2.92)	(1.95)	(1.89)	(1.37)	(-0.52)	(-1.40)	(0.68)	(-1.55)
BLS_delta (%)	7.46	8.37	11.97	11.08	-8.04	-12.70	3.54	-11.13
t-Stat	(2.83)	(1.87)	(1.87)	(1.28)	(-0.82)	(-1.47)	(0.56)	(-1.10)
Panel G: Calls with 2 Month Maturity								
K/S (%)	2.94	4.73	5.55	2.58	-17.10	-14.22	0.35	-13.97
t-Stat	(0.97)	(1.09)	(0.96)	(0.35)	(-2.12)	(-2.34)	(0.07)	(-2.77)
log(K/S)/σ (%)	3.90	3.95	3.32	-0.94	-11.53	-10.66	-1.53	-14.12
t-Stat	(1.29)	(0.84)	(0.57)	(-0.13)	(-1.45)	(-1.74)	(-0.32)	(-2.88)
BLS_delta (%)	3.70	4.10	2.89	-5.90	-23.51	-21.64	-5.38	-23.29
t-Stat	(1.28)	(0.91)	(0.46)	(-0.76)	(-2.84)	(-2.90)	(-0.93)	(-3.49)

Table VII
Relative Call Prices

This table reports time series average relative call prices for different strike call options. We assign each call to a strike group based on its value of K/S, log(K/S)/σ, or BLS-delta, where σ is the annualized stock previous 60 daily return volatility and the volatility in delta. Table III defines the range of K/S, log(K/S)/σ, or BLS-delta for each strike group. Strike group 1has the lowest strike price, and strike group 5 has the highest strike price. The relative option price in strike group G is estimated as:

$$RP_t^{C,G} = \frac{1}{N}\sum_i^N \left(\frac{V_{i,t}}{C_{i,t}} - 1 \right)$$

where i indexes options that belong to strike group G, $C_{i,t}$ is the call mid price, and $V_{i,t}$ is the estimated call price based on equation (11). Panel A computes call relative prices from the midpoint of bid and ask prices, Panel B from ask prices and Panel C from bid prices. Options are selected if all the following conditions are satisfied: (1) the underlying stock does not have an ex-dividend date during remaining life of options, (2) the bid price is larger than $0.125, and (3) call prices are within a no-arbitrage bound. The time period is from January 1996 through June 2005.

Strike group	1	2	3	4	5
Panel A: bid ask mid points					
K/S	0.02	-0.07	-0.15	-0.20	-0.39
t-Stat	(1.35)	(-3.60)	(-4.91)	(-4.44)	(-7.37)
Log(K/S)/σ	-0.03	-0.15	-0.12	-0.16	-0.23
t-Stat	(-1.51)	(-6.31)	(-2.07)	(-4.55)	(-4.98)
BLS_delta	-0.01	-0.09	-0.16	-0.20	-0.38
t-Stat	(-0.45)	(-2.49)	(-4.54)	(-3.97)	(-6.93)
Panel B: ask prices					
K/S	-0.01	-0.11	-0.21	-0.31	-0.49
t-Stat	(-0.26)	(-5.98)	(-7.73)	(-7.78)	(-10.86)
Log(K/S)/σ	-0.06	-0.21	-0.18	-0.24	-0.34
t-Stat	(-3.82)	(-8.97)	(-3.50)	(-7.26)	(-8.53)
BLS_delta	-0.04	-0.13	-0.23	-0.31	-0.48
t-Stat	(-3.65)	(-3.98)	(-7.14)	(-7.33)	(-10.86)
Panel C: bid prices					
K/S	0.05	-0.02	-0.06	-0.03	-0.22
t-Stat	(3.04)	(-1.07)	(-1.68)	(-0.46)	(-3.24)
Log(K/S)/σ	0.01	-0.09	-0.03	-0.06	-0.05
t-Stat	(0.93)	(-3.30)	(-0.43)	(-1.36)	(-0.82)
BLS_delta	0.03	-0.03	-0.06	0.00	-0.18
t-Stat	(2.87)	(-0.83)	(-1.51)	(-0.01)	(-2.36)

Table VIII
Call Returns under Theoretical Call Prices

This table reports averages of monthly time series of the return to holding one month call portfolio to expiration for various strike groups and return differences between strike groups, where the call prices are estimated by four different theoretical models. Each call is assigned to a strike group based on its value of K/S, log(K/S)/σ, or Black-Scholes delta (BLS_delta), where σ is the annualized stock return volatility estimated from the previous 60 daily returns. This is also the volatility used when computing Black-Scholes deltas and call prices. Table III lists the range of K/S, log(K/S)/σ, or BLS_delta for each strike group. Strike Group 1 refers to the lowest strike prices, and group 5 contains the highest strike prices. *X*'-*Y*' is the call return difference between strike group *X* and group *Y* for the same underlying stocks. Call returns are computed based the Equation (8), where the call prices are estimated from Black-Scholes(1972) (Panel A), Merton (1976) jump without jump premia (Panel B), jump adjusted for 'peso' problem (Panel D) and jump with jump premia (Panel D). The physical jump probability λ^P and average jump size κ^P for each stock are estimated from daily returns 3 stand deviations from mean during period from January 1996 to June 2005 when returns are available. Call options are selected on each option expiration date that mature the next expiration date if all the following conditions are satisfied: (1) the underlying stock does not have ex-dividend date during remaining life of the option, (2) the bid price is strictly larger than $0.125, (3) the call market price satisfies a no-arbitrage restriction, and (4) the theoretical option price is at least $0.01. The sample period is from January 1996 to June 2005.

	1	2	3	4	5	5' - 1'	4' - 1'	5' - 2'
Panel A: Black-Scholes Price								
K/S (%)	2.16	4.87	10.17	22.54	63.01	65.98	9.98	69.63
t-Stat	(0.94)	(1.37)	(1.76)	(2.44)	(2.38)	(2.14)	(1.37)	(2.53)
Log(K/S)/σ (%)	4.34	6.23	4.67	6.38	88.25	90.19	3.83	48.88
t-Stat	(1.75)	(1.46)	(0.82)	(0.82)	(5.24)	(5.46)	(0.68)	(3.96)
BLS_delta (%)	4.30	5.58	4.44	10.39	105.29	106.61	6.35	57.52
t-Stat	(1.68)	(1.27)	(0.75)	(1.22)	(5.39)	(5.53)	(1.01)	(3.88)
Panel B: Merton Jump without Jump Premium: $\lambda^Q=\lambda^P$ $\kappa^Q=\kappa^P$								
K/S (%)	2.11	4.66	9.12	18.91	47.41	49.98	6.73	52.54
t-Stat	(0.92)	(1.31)	(1.59)	(2.11)	(2.18)	(2.04)	(0.97)	(2.39)
Log(K/S)/σ (%)	4.25	5.85	3.90	4.36	75.49	76.23	1.76	35.81
t-Stat	(1.72)	(1.38)	(0.69)	(0.57)	(4.88)	(5.10)	(0.32)	(3.28)
BLS_delta (%)	4.24	5.23	3.46	8.07	88.88	89.03	4.09	41.22
t-Stat	(1.66)	(1.19)	(0.59)	(0.97)	(5.21)	(5.36)	(0.66)	(3.56)
Panel C: Jump adjusted for 'Peso': $\lambda^Q=3\lambda^P$ $\kappa^Q=\kappa^P$								
K/S (%)	2.01	4.47	8.56	16.73	53.81	59.68	4.79	41.17
t-Stat	(0.88)	(1.26)	(1.50)	(1.91)	(2.25)	(2.09)	(0.71)	(2.33)
Log(K/S)/σ (%)	4.13	5.60	3.44	3.32	71.50	71.98	0.79	35.58
t-Stat	(1.67)	(1.32)	(0.61)	(0.44)	(4.83)	(5.06)	(0.15)	(3.17)
BLS_delta (%)	4.11	4.97	2.94	6.83	84.01	83.99	2.93	41.57
t-Stat	(1.61)	(1.14)	(0.50)	(0.83)	(5.17)	(5.34)	(0.48)	(3.41)

Table VIII – *Continued*

Panel D: Jump with Jump Premia: $\lambda^Q=2\lambda^P$ $\kappa^Q=\kappa^P$-0.04								
K/S (%)	2.02	4.48	8.68	17.52	60.46	67.79	5.53	46.19
t-Stat	(0.88	(1.26)	(1.52)	(1.98)	(2.31)	(2.14)	(0.81)	(2.39)
Log(K/S)/σ (%)	4.14	5.63	3.53	3.64	74.76	75.51	1.13	38.15
t-Stat	(1.67	(1.33)	(0.63)	(0.48)	(4.96)	(5.18)	(0.21)	(3.28)
BLS_delta (%)	4.11	5.00	3.06	7.23	87.81	88.02	3.34	44.44
t-Stat	(1.61)	(1.14)	(0.52)	(0.87)	(5.29)	(5.45)	(0.55)	(3.50)

Table IX
Frequencies of Large Stock Returns

This table lists the frequencies of large stock returns for the time period from 199601 to 200506 and the period from 196301 to 199512. Panel A are for the frequencies for all stocks, Panel B is for optionable stocks from 199601 to 200506 and their matched stocks before 1996, and Panel C for optionable in strike group 5 and their matched stocks before 1996.

r: Monthly Return	Prob(r>0.15)	Prob(r>0.20)	Prob(r>0.30)
Panel A: All stocks			
All stocks 199601 - 200506	12.23%	8.38%	4.26%
All stocks 196301 - 199512	10.98%	7.05%	3.14%
Panel B: Optionable stocks			
Optionable stocks 199601 -200506	13.91%	8.98%	4.13%
Matched stocks by size decile 196301 -199512	10.01%	5.91%	2.27%
Matched stocks by volume decile 196301 -199512	11.42%	7.26%	3.26%
Matched stocks by volatility decile 196301 -199512	10.43%	6.29%	2.55%
Panel C: Optionable stocks with calls in Strike Group 5			
Optionable stocks at Strike Group 5 199601 -200506	18.03%	12.87%	6.72%
Matched stocks by size decile 196301 -199512	10.67%	6.43%	2.58%
Matched stocks by volume decile 196301 -199512	11.42%	7.36%	3.38%
Matched stocks by volatility decile 196301 -199512	11.91%	7.57%	3.30%

Table X
Idiosyncratic skewness and volatility of five strike groups

This table reports idiosyncratic skewness and volatility of one month call option returns in five strike groups. Idiosyncratic skewness and idiosyncratic volatility are the skewness and standard deviation of call returns. Each call is assigned to a strike group based on its value of K/S, log(K/S)/σ, or BLS-delta, according to the ranges provided in Table III. Strike group 1 is comprised of calls with the lowest strike prices, and group 5 is comprised of calls with the highest strike prices. Panel A reports idiosyncratic skewnesss and volatility based on expected call returns. On each month *t*'s option expiration day, the underlying stock's one month return distribution is estimated from the previous 24-36 monthly returns, from which the expected call return distribution is projected. The expected call return idiosyncratic skewnewss and volatility is computed from this projected distribution. Panel B reports monthly average skewness and volatility of realized call returns in each strike group.

Strike group	1	2	3	4	5
Panel A.1: Expected Idiosyncratic Skewness					
K/S	0.40	0.69	1.61	2.88	3.51
Log(K/S)/σ	0.39	0.95	1.68	2.77	3.49
BLS_delta	0.39	1.01	1.83	2.94	3.50
Panel A.2: Expected Idiosyncratic Volatility (%)					
K/S	50.61	87.94	173.78	306.14	414.42
Log(K/S)/σ	55.56	108.11	177.88	296.81	394.93
BLS_delta	55.57	113.83	192.79	315.52	399.82
Panel B.1: Realized Idiosyncratic Skewness					
K/S	0.80	1.31	2.90	6.00	7.73
Log(K/S)/σ	0.79	1.36	2.79	5.72	7.56
BLS_delta	0.95	1.63	3.04	5.88	7.54
Panel B.2: Realized Idiosyncratic Volatility (%)					
K/S	47.36	77.75	140.20	223.41	229.95
Log(K/S)/σ	49.35	85.78	139.05	219.09	287.40
BLS_delta	56.18	97.93	149.37	224.90	286.57

Table XI
Call Portfolio Returns

This table reports returns on 25 call portfolios sorted independently on expected skewness and expected volatility of each call return. "5-1" refers to return between portfolio 5 and portfolio 1. On each month t's option expiration day, we estimate underlying stock's one month return distribution using the previous 24-36 monthly returns, from which we project expected call return distribution and then generate expected call return skewness and volatility. Panel A reports each portfolio's average returns in percentage, and Panel B reports the standard t-test of each return.

	1 low skew	2	3	4	5 high skew	5-1
Panel A: Returns (%)						
1 low σ	-1.91	1.36	0.72	-8.21	-44.44	-42.53
2	3.37	2.87	1.34	-3.79	-24.22	-27.58
3	2.74	3.05	0.66	-3.64	-18.17	-20.91
4	13.08	8.82	3.00	-2.38	-13.45	-22.29
5 high σ	11.43	23.74	7.66	3.66	-7.39	-21.62
5-1	23.54	22.81	6.95	11.88	37.05	
Panel B: Returns *t*-Stat						
1 low σ	(-1.02)	(0.77)	(0.41)	(-3.04)	(-7.88)	(-9.13)
2	(1.17)	(1.03)	(0.48)	(-1.19)	(-4.07)	(-5.48)
3	(0.69)	(0.80)	(0.17)	(-0.92)	(-3.83)	(-6.60)
4	(1.60)	(1.61)	(0.59)	(-0.46)	(-2.51)	(-3.80)
5 high σ	(1.49)	(1.69)	(1.05)	(0.51)	(-0.91)	(-2.68)
5-1	(1.86)	(1.72)	(1.16)	(2.01)	(6.43)	

Endnotes

[1] Results similar to those presented below are obtained if none of the first three conditions are imposed.

[2] The beta of a stock is computed as the slope coefficient from a regression of the previous 24 monthly stock returns on market returns.

[3] Volatility is the annualized sample estimate from the previous 60 daily stock returns.

[4] The reason that the change in return can be different in group5 than groups 1-4 is that the universe of underlying stocks for group 5 is different.

[5] There is some evidence that co-skewness impacts the returns of securities (Kraus and Litzenberger (1976) and Harvey and Siddique (2000).) Co-skewness effects, however, are consistent with globally concave utility and so are not natural candidates for explaining the violations of the call return restrictions.